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Dear Students,

This Institute is dedicated to cater the needs of students preparing for Indian Statistical Service. We publish videos on Youtube channel for student help.

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TEST SERIES PAPER-2 (TEST-1)

SOLUTION

Q-1 ⇒ $X \sim P(\lambda) \Rightarrow P(x) = \frac{e^{-\lambda}\cdot \lambda^x}{x!}$; $x = 0, 1, 2, \dots$

Since $\delta(x)$ is U.E of $g(\lambda)$

$\Rightarrow E(\delta(x)) = g(\lambda)$

$\Rightarrow \sum_{x=0}^{\infty} \delta(x)\cdot \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}[3\lambda^2+2\lambda+1]$

$\Rightarrow \delta(0) + \lambda\cdot\delta(1) + \frac{\lambda^2}{2!}\delta(2) + \sum_{x=3}^{\infty} \delta(x)\cdot\frac{\lambda^x}{x!} = 3\lambda^2+2\lambda+1$

$\Rightarrow \delta(0) = 1$; $\delta(1) = 2$; $\delta(2) = 6$; $\delta(x) = 0 \;\forall\; x \geq 3$

$\Rightarrow \sum_{x=0}^{\infty} \delta(x) = 9 \Rightarrow$ a-option.

Q-2 ⇒ $X_1, X_2, X_3 \sim U(0, \theta) \Rightarrow E(X_i) = \frac{\theta}{2}$

$E(T) = \theta \Rightarrow \frac{3\theta + 2\theta + a\theta}{6} = \theta \Rightarrow a = 1$

⇒ a-option

Q-3 ⇒ Here $f(x) = \frac{1}{2} e^{-|x-\theta|} \;\forall\; -\infty < x < \infty$

$L = \frac{1}{2^n}\cdot e^{-\sum|x_i-\theta|}$

⇒ L is Maximum then $\sum|x_i-\theta|$ is Minimum

w.r.t. to $\theta \Rightarrow \hat{\theta}_{MLE}$ = Median of Sample.

⇒ c-option.

Q-4 $\Rightarrow$ $X \sim \exp(\theta)$ $\Rightarrow$ $E(X) = \frac{1}{\theta}$; $V(X) = \frac{1}{\theta^2}$

$\Rightarrow$ $E(\bar{X}) = \frac{1}{\theta}$ and $V(\bar{X}) = \frac{1}{n\theta^2}$.

$\Rightarrow$ For Large n :-

$$\frac{\bar{X} - \frac{1}{\theta}}{\frac{1}{\theta} \cdot \frac{1}{\sqrt{n}}} \sim N(0,1)$$

$$\Rightarrow P\left[-1.96 \leq \frac{\bar{X} - \frac{1}{\theta}}{\frac{1}{\theta} \cdot \frac{1}{\sqrt{n}}} \leq 1.96\right] = 0.95$$

$$\Rightarrow P\left[\left(1 - \frac{1.96}{\sqrt{n}}\right) \cdot \frac{1}{\bar{X}} \leq \theta \leq \left(1 + \frac{1.96}{\sqrt{n}}\right) \cdot \frac{1}{\bar{X}}\right] = 0.95$$

$\Rightarrow$ b-option

Q-5 $\Rightarrow$ $X \sim \exp\left(\frac{1}{\theta}\right)$; $Y \sim \exp\left(\frac{1}{2\theta}\right)$; $Z \sim \exp\left(\frac{1}{3\theta}\right)$

$$\Rightarrow L = \frac{1}{\theta} \cdot e^{-x/\theta} \cdot \frac{1}{2\theta} e^{-y/2\theta} \cdot \frac{1}{3\theta} e^{-z/3\theta}$$

$$\Rightarrow L = \frac{1}{6\theta^3} e^{-\frac{1}{\theta}\left(x + \frac{y}{2} + \frac{z}{3}\right)}$$

$$\Rightarrow \log L = -\log 6 - 3\log\theta - \frac{1}{\theta}\left(x + \frac{y}{2} + \frac{z}{3}\right)$$

$$\Rightarrow \frac{\partial \log L}{\partial \theta} = -\frac{3}{\theta} + \frac{1}{\theta^2}\left(x + \frac{y}{2} + \frac{z}{3}\right) = 0$$

$$\Rightarrow \hat{\theta} = \frac{1}{3}\left(x + \frac{y}{2} + \frac{z}{3}\right)$$

$\Rightarrow$ c-option.

Q-6 ⇒ $X \sim b(5, \theta) \Rightarrow E(X) = 5\theta$; $V(X) = 5\theta(1-\theta)$

$$\Rightarrow E(X^2) = V(X) + E^2(X) = 5\theta(1-\theta) + 25\theta^2$$

$$\Rightarrow E\left(\frac{X}{5}\right) = \theta$$

Consider $\left(\frac{X}{5}\right)\left(1-\frac{X}{5}\right)$ for $\theta(1-\theta)$.

so

$$E\left[\frac{X}{5}\left(1-\frac{X}{5}\right)\right] = E\left[\frac{X(5-X)}{25}\right] = \frac{1}{25}\left[5E(X) - E(X^2)\right]$$

$$= \frac{1}{25}\left[25\theta - 5\theta + 5\theta^2 - 25\theta^2\right] = \frac{1}{25}\left[20\theta - 20\theta^2\right]$$

$$\Rightarrow E\left[\frac{X(5-X)}{20}\right] = \theta(1-\theta)$$

Since X is S.S for θ

$\Rightarrow \frac{5X - X^2}{20}$ is U.M.V.U.E for $\theta(1-\theta)$.

⇒ a- option

Q-7 ⇒ first we define density f^n as

$$f(x) = \frac{1}{3}\cdot\theta\cdot e^{-\theta x} + \frac{1}{4}\,20\,e^{-20x} + \frac{5}{12}\cdot 30\,e^{-30x}.$$

$$\Rightarrow E(X) = \frac{1}{3}\cdot\frac{1}{\theta} + \frac{1}{4}\cdot\frac{1}{20} + \frac{5}{12}\cdot\frac{1}{30} = \frac{43}{72}\cdot\frac{1}{\theta}$$

by MoME theory $E(X) = \bar{X}$

$$\Rightarrow \hat{\theta}_{MoME} = \frac{43}{72}\cdot\frac{1}{\bar{X}}$$

⇒ d-option

Q-8 ⇒ $f(x) = \frac{1}{\theta\sqrt{2\pi}} e^{-x^2/2\theta^2}$

$\Rightarrow L = \frac{1}{\theta^n}\left(\frac{1}{2\pi}\right)^{n/2} e^{-\frac{1}{2\theta^2}\sum x_i^2}$

$\Rightarrow \log L = -n\log\theta - \frac{n}{2}\log 2\pi - \frac{1}{2\theta^2}\sum x_i^2$

$\Rightarrow \frac{\partial \log L}{\partial\theta} = -\frac{n}{\theta} + \frac{1}{\theta^3}\sum x_i^2 = 0$

$\Rightarrow \hat{\theta}_{MLE} = \sqrt{\frac{\sum x_i^2}{n}}$ ⇒ d- option

Q-9 ⇒ $f(x) = \frac{x}{\theta^2}\cdot e^{-x/\theta}$

$\Rightarrow L = \prod_{i=1}^{n} x_i \cdot \theta^{-2n} e^{-\frac{1}{\theta}\sum x_i}$

$\Rightarrow \log L = -2n\log\theta + \sum \log x_i - \frac{1}{\theta}\sum x_i$

$\Rightarrow \frac{\partial \log L}{\partial\theta} = -\frac{2n}{\theta} + \frac{1}{\theta^2}\sum x_i = 0 \Rightarrow \hat{\theta} = \frac{1}{2n}\sum x_i$

⇒ c- option

Q-10 ⇒ $X \sim U(1,\theta)$; $\theta > 1$ ⇒ $f_x(x) = \frac{1}{\theta-1}$

$\Rightarrow F(x) = \frac{x-1}{\theta-1}$.

Here $X_{(n)} = \max\{X_i\}$ is S.S for θ

let $Y = X_{(n)}$ ⇒ $f_y(y) = n[F_X(y)]^{n-1} f_X(y)$

$\Rightarrow f_y(y) = n\left(\frac{y-1}{\theta-1}\right)^{n-1}\cdot\frac{1}{\theta-1}$

Now
$$E(X_{(n)}-1) = E(Y-1) = \int_{y=1}^{\theta} (y-1)\cdot\frac{n}{(\theta-1)^n}(y-1)^{n-1}dy$$

$$= \frac{n}{(\theta-1)^n}\int_{y=1}^{\theta}(y-1)^n dy = \frac{n}{(\theta-1)^n}\cdot\left[\frac{(y-1)^{n+1}}{n+1}\right]_1^{\theta}$$

$$= \frac{n}{(\theta-1)^n}\cdot\frac{1}{(n+1)}\cdot\left[(\theta-1)^{n+1}\right] = \frac{n}{n+1}\cdot(\theta-1)$$

$$\Rightarrow E(Y)-1 = \frac{n\theta}{n+1}-\frac{n}{n+1}$$

$$\Rightarrow E\left[Y-1+\frac{n}{n+1}\right] = \frac{n\theta}{n+1}$$

$$\Rightarrow E\left[(n+1)\cdot Y - n - 1 + n\right] = n\theta$$

$$\Rightarrow E\left[\frac{n+1}{n}\cdot X_{(n)} - \frac{1}{n}\right] = \theta$$

$\Rightarrow \frac{n+1}{n}X_{(n)} - \frac{1}{n}$ is U.M.V.U.E of θ

$\Rightarrow$ b-option

Q-11 $\Rightarrow$ $X \sim U(\theta, \theta+5) \Rightarrow \theta \le X \le \theta+5$

$$\Rightarrow \theta \le X_{(1)} < X_{(2)} < \cdots < X_{(n)} \le \theta+5$$

$$\Rightarrow X_{(n)}-5 \le \theta \le X_{(1)}$$

According to observed sample:

$$7.5-5 \le \hat{\theta} \le 3.5$$

$$\Rightarrow 2.5 \le \hat{\theta} \le 3.5$$

$\Rightarrow 2.4 \notin [2.5, 3.5] \Rightarrow 2.4$ is Not M.L.E

$\Rightarrow$ a-option

Q-12 ⇒ $f(x) = \frac{1}{2} e^{-|x-\theta|}$; $-\infty < x < \infty$

⇒ $L = \frac{1}{2^n} \cdot e^{-\Sigma|x_i - \theta|} \neq g_1(T, \theta) \cdot g_2(x_i)$

⇒ S.E for θ does not exist. ⇒ d-option

Q-13 ⇒ $X \sim U(\theta-1, \theta+1)$

⇒ $\theta-1 \leq X_{(1)} < X_{(2)} < \cdots < X_{(n)} \leq \theta+1$.

⇒ $X_{(n)} - 1 \leq \hat{\theta} \leq X_{(1)} + 1$

⇒ $\{X_{(1)}, X_{(n)}\}$ is Jointly S.S for θ and $\{X_{(n)}-1, X_{(1)}+1\}$ is also Jointly S.S for θ.

⇒ c- option

Q-14 ⇒ $f(x) = \frac{x}{\theta^2} \cdot e^{-x/\theta}$: $x > 0$; $\theta > 0$.

⇒ $L = \frac{\prod_{i=1}^{n} x_i}{\theta^{2n}} \cdot e^{-\frac{1}{\theta}\Sigma x_i} = \frac{1}{\theta^{2n}} e^{-\frac{1}{\theta}\Sigma x_i} \cdot \prod_{i=1}^{n} x_i$

⇒ $\hat{\theta} = \sum_{i=1}^{n} x_i$ is S.S for θ. ⇒ a- option

Q-15 ⇒ $f(x) = \frac{1}{2} \cdot e^{-|x-\theta|}$; $-\infty < x < \infty$

⇒ $L = \left(\frac{1}{2}\right)^n e^{-\Sigma|x_i - \theta|}$

⇒ L is Maximum if $\Sigma|x_i - \theta|$ is minimum

⇒ $\hat{\theta}$ = median ⇒ b-option

Q-16 $\Rightarrow$ $X \sim N(\mu, 2) \Rightarrow E(X) = \mu$; $V(X) = 2$

$\Rightarrow E(X^2) = 2 + \mu^2$

Now $E(T) = \frac{1}{n} \sum_{i=1}^{n} E(X_i^2) = E(X^2) = 2 + \mu^2$

$\Rightarrow$ d-option

Q-17 $\Rightarrow$ All of above. $\Rightarrow$ d-option

Q-18 $\Rightarrow$ $X \sim P(\mu) \Rightarrow E(X) = \mu$ and $V(X) = \mu$

$\Rightarrow E(T) = \mu$ and $V(T) = \frac{\mu}{n}$

$\Rightarrow M.S.E = V(T) + (\text{bias } T)^2 = \frac{\mu}{n} + 0 = \frac{\mu}{n}$

$\Rightarrow$ d-option

Q-19 $\Rightarrow$ $P(x) = p^x (1-p)^{1-x}$

$\Rightarrow L = p^{\Sigma x_i} (1-p)^{n - \Sigma x_i} \cdot 1$

$\Rightarrow T = \Sigma x_i$ is S.S for p. $\Rightarrow$ c-option.

Q-20 $\Rightarrow$ $S.E(p) = \sqrt{\frac{PQ}{n}}$ $\Rightarrow$ d-option

Q-21 $\Rightarrow$ I and II are statistics because they are function of sample observations and constant (σ^2). not the function of parameter μ. $\Rightarrow$ a-option.

Q-22 ⇒ $X \sim b(1, p)$; $p \in [\frac{1}{7}, \frac{4}{7}]$

⇒ $L = p^x (1-p)^{1-x}$; $x = 0, 1.$

if $x = 0$ then $L = 1-p$ ⇒ L is max at $p = \frac{1}{7}$

if $x = 1$ then $L = p$ ⇒ L is max at $p = \frac{4}{7}$

⇒ $T = \frac{3X+1}{7}$ gives same result at $x = 0, 1.$

⇒ $\hat{p}_{MLE} = \frac{3X+1}{7}$ ⇒ c- option.

Q-23 ⇒ $E(X_1) = \mu$ and $E(X_2) = 2\mu.$

$V(X_1) = \sigma^2$ and $V(X_2) = \sigma^2.$

Since ε_1 and ε_2 are Independent so X_1 and X_2 are also Independent.

Here $T_1 = \frac{2X_1 + X_2}{3}$ and $T_2 = \frac{X_1 + 2X_2}{5}$

are only the U.E for μ.

⇒ $V(T_1) = \frac{5\sigma^2}{9}$ and $V(T_2) = \frac{5\sigma^2}{25}$

Since $V(T_2) < V(T_1)$

⇒ $\frac{X_1 + 2X_2}{5}$ is B.L.U.E for μ.

⇒ b - option.

Q-24 ⇒ $X \sim b(5, \theta)$ ⇒ $E(X) = 5\theta$; $V(X) = 5\theta(1-\theta)$

⇒ $E(X^2) = 5\theta - 5\theta^2 + 25\theta^2 = 5\theta + 20\theta^2$

⇒ $\theta = E\left(\frac{X}{5}\right)$; $\theta^2 = E\left(\frac{X^2}{20}\right) - E\left(\frac{X}{20}\right)$

Now

$\theta(1+\theta) = \theta + \theta^2 = E\left(\frac{X}{5}\right) + E\left(\frac{X^2}{20}\right) - E\left(\frac{X}{20}\right)$

⇒ $\theta(1+\theta) = E\left[\frac{X(X+3)}{20}\right]$

Since X is SS for θ

⇒ $\frac{X(X+3)}{20}$ is UMVUE for $\theta(1+\theta)$.

⇒ c-option

Q-25 ⇒ $E(X_1) = \mu$; $V(X_1) = \sigma^2$

$E(X_2) = \mu$; $V(X_2) = \sigma^2$

⇒ $V(T_1) = \frac{1}{9} \times 5\sigma^2 = \frac{5\sigma^2}{9}$

$V(T_2) = \frac{\sigma^2}{2}$

⇒ $E_f(T_1/T_2) = \frac{V(T_2)}{V(T_1)} = \frac{9}{10}$ ⇒ d-option

Q-26 ⇒ $\hat{p}_1 = \frac{20}{100}$; $\hat{p}_2 = \frac{13}{50}$

⇒ $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2} = \frac{20+13}{150}$ ⇒ b-option

Q-27 ⇒ $P[50 \le \mu \le 60] = 0.95$

$\Rightarrow P[52 \le \mu \le 58] < P[50 \le \mu \le 60]$

$\Rightarrow P[52 \le \mu \le 58] \neq 0.99 \Rightarrow$ a-option

Q-28 ⇒ $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

$\Rightarrow L = \frac{1}{\sigma^n} \cdot \frac{1}{(2\pi)^{n/2}} \cdot e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}$

$\Rightarrow \log L = -n\log\sigma - \frac{n}{2}\log 2\pi - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$

$\Rightarrow \frac{\partial \log L}{\partial \mu} = -\frac{1}{2\sigma^2} \times (-2)\sum(x_i-\mu) = 0 \Rightarrow \hat{\mu} = \bar{x}$

$\frac{\partial \log L}{\partial \sigma} = -\frac{n}{\sigma} - \frac{1}{2}\sum(x_i-\mu)^2 \times (-2\cdot\sigma^{-3}) = 0$

$\Rightarrow \hat{\sigma} = \sqrt{\frac{1}{n}\sum(x_i-\bar{x})^2}$

⇒ b- option.

Q-29 ⇒ $X \sim P(\theta) \Rightarrow E(X) = \theta$

$\Rightarrow E(T) = n\theta$

⇒ T is biased Estimator for θ.

⇒ b- option

Q-30 ⇒ Since T is CE of $\theta \Rightarrow E(T) = \theta$ and $V(T) = 0$ as $n \to \infty$.

let us Consider $T' = T + \frac{1}{n}$

$\Rightarrow \lim_{n\to\infty} E(T') = \lim_{n\to\infty} E(T) + \lim_{n\to\infty} \frac{1}{n} = \theta$

Now $\lim_{n\to\infty} V\left(T+\frac{1}{n}\right) = \lim_{n\to\infty} V(T) = 0$

$\Rightarrow T' = T + \frac{1}{n}$ is C.E for θ. $\Rightarrow$ b-option

Q-31 $\Rightarrow V(T_1) = V(X_1+X_2-X_3) = 3\sigma^2$ $\Rightarrow$ b-option

Q-32 $\Rightarrow V(T_1) = 3\sigma^2$

and $V(\bar{x}) = \sigma^2/3$

$\Rightarrow E_f(T_1/\bar{x}) = \frac{\sigma^2}{3} \times \frac{1}{3\sigma^2} = \frac{1}{9}$ $\Rightarrow$ c-option.

Q-33 $\Rightarrow x \sim P(\lambda) \Rightarrow P(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}$

Since $\delta(x)$ is U.E of $e^{-\lambda}$. $\Rightarrow E(\delta(x)) = e^{-\lambda}$.

$\Rightarrow \sum_{x=0}^{\infty} \delta(x)\cdot \frac{e^{-\lambda}\cdot\lambda^x}{x!} = e^{-\lambda}$.

$\Rightarrow \delta(0) + \delta(1)\lambda + \sum_{x=2}^{\infty} \delta(x)\cdot\frac{\lambda^x}{x!} = 1 + 0\cdot\lambda + 0\cdot\lambda^2 + \cdots\cdots$

Compare both side.

$\Rightarrow \delta(0) = 1$; $\delta(x) = 0 \ \forall \ x \geq 1$

$\Rightarrow \sum_{K=0}^{\infty} \delta(K) = 1$.

$\Rightarrow$ c-option

Q-34 ⇒ $X \sim P(\lambda) \Rightarrow P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$\Rightarrow L = \frac{e^{-n\lambda} \cdot \lambda^{\Sigma x_i}}{\Pi x_i!}$

$\Rightarrow \log L = -n\lambda + \Sigma x_i \log \lambda - \Sigma \log x_i!$

$\Rightarrow \frac{\partial \log L}{\partial \lambda} = -n + \frac{\Sigma x_i}{\lambda} = \frac{\bar{x} - \lambda}{\lambda / n}$

⇒ $\bar{x}$ is M.V.B.U.E of λ and $V(\bar{x}) = \frac{\lambda}{n}$

⇒ b- option.

Q-35 ⇒ $X \sim N(\mu, \mu) \Rightarrow E(X) = \mu$ and $V(X) = \mu$

$\Rightarrow E(X^2) = \mu + \mu^2$

$\Rightarrow E(T) = \frac{1}{n} \sum_{i=1}^{n} E(X_i^2) = \frac{1}{n} \cdot n \cdot E(x^2) = \mu(\mu + 1)$

⇒ c- option.

Q-36 ⇒ Since 98% C.I is (20.49, 23.51) which contains the true value of $\mu = 20.5$. So we Accept the null Hypothesis at 2% level of Significance. But as level of Significance will increase the confidence Interval will shrink. So may be Rejected at 5% as well as 10% level of Significance.
⇒ c- option.

Q-37 ⇒ p = Sample proportion $= \frac{59}{100} = 0.59$.

$H_0 : P = 0.5$

$H_A : P \neq 0.5$.

The Test statistic is $Z = \dfrac{p-P}{\sqrt{\frac{P.Q}{n}}}$, under H_0

$\Rightarrow Z = \dfrac{0.59 - 0.50}{\sqrt{\frac{1}{2}\cdot\frac{1}{2}/100}} = 1.80 \Rightarrow$ a-option

Q-38 ⇒ $f(x) = \frac{1}{\lambda}\cdot e^{-x/\lambda}$; $\lambda > 0$. ; $H_0 : \lambda = 3$, $H_1 : \lambda = 5$

$W_c = \{x : x \geq 4.5\}$

$\alpha = P[X \geq 4.5 / H_0] = \int_{4.5}^{\infty} \frac{1}{3} e^{-x/3} dx = e^{-1.5} = 0.2231$

Power $= P[X \geq 4.5 / H_1] = \int_{4.5}^{\infty} \frac{1}{5} e^{-x/5} dx = e^{-0.9} = 0.4065$

⇒ d- option

Q-39 ⇒ let X denote no of Success. ⇒ $X \sim b(5, p)$

$H_0 : p = \frac{1}{2}$

$H_1 : p = \frac{3}{4}$

$\Rightarrow W_c = \{x ; x > 3\}$

$\beta = P[X \leq 3 / H_1] = 1 - P[X > 3 / H_1]$

$= 1 - P[X = 4, 5 / p = \frac{3}{4}]$

$= 1 - \left[{}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) + {}^5C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0 \right] = \frac{47}{128}$

⇒ d- option.

Q-40 ⇒ $X \sim b(3, p)$ ⇒ X: 0, 1, 2, 3

$P(x) = {}^3C_x \cdot p^x q^{3-x}$; $q = 1-p$

$W_c = \{X : X \geq 2\}$

$H_0 : p = 2/3$

$H_1 : p = 1/3$

$\alpha = P[X \geq 2 / H_0]$

$= P[X = 2, 3 / p = 2/3]$

$= {}^3C_2 \cdot \frac{4}{9} \cdot \frac{1}{3} + {}^3C_3 \cdot \frac{8}{27} \cdot 1 = \frac{20}{27}$.

$\beta = P[X \leq 1 / H_1] = P[X = 0, 1 / p = \frac{1}{3}]$

$= {}^3C_0 \cdot p^0 q^3 + {}^3C_1 \, p \cdot q^2$

$= 1 \cdot 1 \cdot \frac{8}{27} + 3 \cdot \frac{1}{3} \cdot \frac{4}{9} = \frac{20}{27}$

⇒ a - option

Q-41 ⇒ First we find Best C.R

$W_c = \{X : \frac{f_0}{f_1} \leq k\} = \{X : \frac{2x}{3x^2} \leq k\} = \{x : x \geq c\}$

Since $\alpha = 0.19$

$\Rightarrow 0.19 = P[X \geq c / f_0] = \int_c^1 2x\,dx = 1 - c^2 \Rightarrow c = 0.9$

Now

Power $= P[X \geq 0.9 / f_1] = \int_{0.9}^1 3x^2 dx = 0.271$

⇒ b - option.

Q-42 ⇒ $X \sim N(\mu, 1)$; $H_0 : \mu = 0$
$H_1 : \mu = \frac{1}{2}$

$W_c = \{ \underset{\sim}{x} \in R^n : \sum_{i=1}^{n} x_i > c \}$; $\alpha = 0.025$; $1-\beta = 0.7054$

$$\Rightarrow \sum_{i=1}^{n} X_i \sim N(n\mu, n)$$

$$\Rightarrow \alpha = 0.025 = P[\Sigma X_i > c / \mu = 0] = P\left[\frac{\Sigma X_i - 0}{\sqrt{n}} > \frac{c}{\sqrt{n}}\right]$$

$$= P\left[Z > \frac{c}{\sqrt{n}}\right]$$

$\Rightarrow \frac{c}{\sqrt{n}} = 1.96$ by normal table. —— (i)

and Power $= 0.7054 = P[\Sigma X_i > c / \mu = \frac{1}{2}] = P\left[\frac{\Sigma X_i - n/2}{\sqrt{n}} > \frac{c - \frac{n}{2}}{\sqrt{n}}\right]$

$$\Rightarrow 0.7054 = P\left[Z > \frac{c - n/2}{\sqrt{n}}\right]$$

$\Rightarrow \frac{c - n/2}{\sqrt{n}} = -0.54$ —— (ii)

by Solving (i) and (ii) we get $n = 25$.

⇒ b-option

Q-43 ⇒ $P(X = k) = p \cdot (1-p)^k$; $k = 0, 1, 2, 3, \cdots$

$H_0 : p = \frac{1}{2}$

$H_1 : p \neq \frac{1}{2}$

$W_c = \{ x : X \leq A \text{ or } X \geq B \}$; A and B are Positive Integers.

$\alpha = P[X \leq A \text{ or } X \geq B]$

$= P[X = 0, 1, 2, \cdots A, B, B+1, \cdots / p = \frac{1}{2}]$

$$\Rightarrow \alpha = \sum_{x=0}^{A} \left(\tfrac{1}{2}\right)\cdot\left(\tfrac{1}{2}\right)^x + \sum_{x=B}^{\infty}\left(\tfrac{1}{2}\right)\cdot\left(\tfrac{1}{2}\right)^x$$

$$= \frac{1}{2}\left[1+\frac{1}{2}+\cdots+\left(\frac{1}{2}\right)^A\right] + \left(\frac{1}{2}\right)^{B+1}\left[1+\frac{1}{2}+\cdots\cdots\right]$$

$$= \frac{1}{2}\left[\frac{1-\left(\frac{1}{2}\right)^{A+1}}{1-\frac{1}{2}}\right] + \left(\frac{1}{2}\right)^{B+1}\cdot\frac{1}{1-\frac{1}{2}} = 1 - 2^{-A-1} + 2^{-B}$$

$\Rightarrow$ c-option

Q-44 $\Rightarrow$ $f(x) = \theta\alpha e^{-\alpha x} + (1-\theta)2\alpha e^{-2\alpha x}$; $x \geq 0$

$H_0: \theta = 1$; $\alpha = 1$.

$H_1: \theta = 0$; $\alpha = 2$.

$$W_c = \left\{ \underset{\sim}{X} : \frac{L_0}{L_1} \leq k \right\} = \left\{ \underset{\sim}{X} : \frac{e^{-\sum x_i}}{4^n \cdot e^{-4\sum x_i}} \leq k \right\}$$

$$= \left\{ \underset{\sim}{X} : e^{3\sum x_i} \leq k' \right\}$$

$\Rightarrow W_c = \left\{ \underset{\sim}{X} : \sum x_i \leq c \right\}$ where c is some positive no i.e $0 < c < \infty$

$\Rightarrow$ c-option

Q-45 $\Rightarrow$ $X \sim b(3, p)$

$W_c = \{ x : x = 3 \}$

$H_0: p = \frac{1}{3}$ and $H_1: p = \frac{2}{3}$

$$\alpha = P[X = 3 / p = \tfrac{1}{3}] = {}^3C_3 \left(\tfrac{1}{3}\right)^3\left(\tfrac{2}{3}\right)^0 = \tfrac{1}{27}$$

$$\beta = P[X = 0,1,2 / p = 2/3] = 1 - P[X = 3/p = 2/3]$$

$$= 1 - \frac{8}{27} = \frac{19}{27}. \quad \Rightarrow \text{c-option}$$

Q-46 $\Rightarrow$ $f(x) = \lambda e^{-\lambda x}$; $x > 0$.

$H_0 : \lambda = 3$
$H_1 : \lambda = 5$
$W_c = \{X : X \le 4.5\}$

$\Rightarrow \alpha = P[X \le 4.5 / \lambda = 3] = \int_0^{4.5} 3 \cdot e^{-3x} dx = 1 - e^{-13.5}$

$\beta = 1 - P[X \le 4.5 / \lambda = 5] = 1 - \int_0^{4.5} 5 \cdot e^{-5x} dx = e^{-22.5}$

$\Rightarrow$ a - option

Q-47 $\Rightarrow$ $f_0(x) = 3x^2$; $0 < x < 1$
$f_1(x) = 4x^3$; $0 < x < 1$.

$H_0 : f = f_0$
$H_1 : f = f_1$

$W_c = \left\{x : \frac{3x^2}{4x^3} \le k\right\} = \{x : x \ge c\}$; $c > 0$.

Since $\alpha = 0.271 = P[X \ge c / f_0] = \int_c^1 3x^2 dx = 1 - c^3$

$\Rightarrow c = 0.9$.

Now Power $= P[X \ge 0.9 / f_1] = \int_{0.9}^1 4x^3 dx = 1 - (0.9)^4 = 0.3439$

$\Rightarrow$ c - option

Q-48 $\Rightarrow$ $f(x) = \theta \cdot x^{\theta - 1}$; $0 < x < 1$

$H_0 : \theta = 3$
$H_1 : \theta = 2$.

$W_c = \{X : X \le 0.5\}$

$\Rightarrow$ Power $= P[X \le 0.5/\theta=2] = \int_0^{0.5} 2x\,dx = 0.25$

$\Rightarrow$ a-option

Q-49 $\Rightarrow$ $X \sim N(\mu,1)$ $\Rightarrow$ $T = \sum_{i=1}^{n} X_i \sim N(n\mu, n)$

$H_0 : \mu = 0$

$H_1 : \mu = 1$

$W_c = \{ \underset{\sim}{x} ; |\sum_{i=1}^{n} x_i| > 1.96 \}$; $n = 25$.

$\alpha = P[|\Sigma X_i| > 1.96/\mu=0] = P[T > 1.96 \text{ or } T < -1.96/\mu=0]$

$= P[\frac{T-0}{\sqrt{n}} > \frac{1.96}{5} \text{ or } \frac{T-0}{\sqrt{n}} < \frac{-1.96}{5}]$

$= P[|Z| > 0.392]$

$= 0.6966$ (by $N(0,1)$ table)

$\Rightarrow$ d- option

Q-50 $\Rightarrow$ $f(x) = \frac{1}{\theta}$; $0 < x < \theta$

$W_c = \{x : x \geq 0.5\}$

$H_0 : \theta = 1$

$H_1 : \theta = 2$.

$\beta = P[X < 0.5/\theta=2] = \int_0^{0.5} \frac{1}{2}\,dx = \frac{1}{4}$

$\Rightarrow$ b-option

Q-51 ⇒
$$Y_1 = \beta_1 + \beta_2 + e_1$$
$$Y_2 = \beta_1 - \beta_2 + \beta_3 - e_2 \quad \Rightarrow \quad \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ -e_2 \\ e_3 \end{bmatrix}$$
$$Y_3 = \beta_1 - \beta_2 + e_3$$

let $\underset{\sim}{l}'\underset{\sim}{\beta} = l_1\beta_1 + l_2\beta_2 + l_3\beta_3$ then $\underset{\sim}{l}'\underset{\sim}{\beta}$ is Estimable

iff $\exists\ (c_1, c_2, c_3)$ s.t $\underset{\sim}{c}'X = \underset{\sim}{l}'$

$$\Rightarrow (c_1, c_2, c_3) \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} = (l_1, l_2, l_3)$$

$$\Rightarrow \left.\begin{array}{l} c_1 + c_2 + c_3 = l_1 \\ c_1 - c_2 - c_3 = l_2 \end{array}\right\} 2c_1 = l_1 + l_2 \Rightarrow c_1 = \frac{l_1 + l_2}{2}$$
$$c_2 = l_3$$

put c_1 and c_2 in Second Eq^n. we get

$$c_3 = \frac{l_1 - l_2 - 2l_3}{2}$$

$\Rightarrow \forall\ (l_1, l_2, l_3),\ \exists\ (c_1, c_2, c_3)$ s.t $\underset{\sim}{c}'X = \underset{\sim}{l}'$

⇒ all parametric functions $\underset{\sim}{l}'\underset{\sim}{\beta}$ are estimable.

⇒ d- option

Q-52 ⇒ g-Inverse of a matrix is always exist.

g-Inverse of a Square Matrix is Not always Unique.

⇒ d- option

Q-53 ⇒ $y_i = \beta_0 + \beta_1 x_i + e_i \quad \forall\ i = 1, 2, \cdots n.$

$$\Rightarrow \sum_{i=1}^{n} y_i = n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i \quad \text{— (i)}$$

$$\sum_{i=1}^{n} x_i y_i = \beta_0 \sum_{i=1}^{n} x_i + \beta_1 \sum_{i=1}^{n} x_i^2 \quad \text{— (ii)}$$

Eq^n (i) and (ii) are normal Equations for O.L.S.

$\Rightarrow \sum y_i = n\beta_0 + n\beta_1$; $x_j = 1 \ \forall\ j$ i.e $\sum_{i=1}^{n} x_i = n$

$\Rightarrow \bar{\beta}_0 + \bar{\beta}_1 = \bar{y}.$

⇒ c- option

Q-54 ⇒ only statement - I is Correct

⇒ a- option

Q-55 ⇒ O.L.S estimators are B.L.U.E, if Usual assumptions are Valid. and

$E(\hat{\beta}_0) = \beta_0$ and $E(\hat{\beta}_1) = \beta_1 \Rightarrow \lim_{n \to \infty} E(\hat{\beta}_1) = \beta_1$

⇒ b-option

Q-56 ⇒ $E(s_t^2) = E(s_E^2) = \sigma^2$ only Under H_0.

otherwise $E(s_t^2) > E(s_E^2)$

$\Rightarrow E(s_t^2) \geq E(s_E^2)$

⇒ d-option

Q-57 ⇒ $\frac{s_t^2}{s_E^2} \sim F(K-1, N-K)$ ⇒ a-option

Q-58 ⇒ y_{iJ}'s are NOT Independent but they are Identical. ⇒ d-option

Q-59 ⇒ $Cov(y_{iJ}, y_{iJ'}) = \sigma_a^2$

⇒ c-option

Q-60 ⇒ $M.S.E(s_E^2)$ is always an U.E for σ_e^2.
⇒ a-option

Q-61 ⇒ a-option

Q-62 ⇒ d-option

Q-63 ⇒ a-option

Q-64 ⇒ d-option

Q-65 ⇒ b-option

Q-66 ⇒ b-option

Q-67 ⇒ b-option

Q-68 ⇒ b-option

Q-69 ⇒ b-option

Q-70 ⇒ d-option

TEST SERIES PAPER-2 (TEST-1) SOLUTION

Q-71 ⇒ b-option

Q-72 ⇒ c-option

Q-73 ⇒ a-option

Q-74 ⇒ d-option

Q-75 ⇒ b-option

Q-76 ⇒ d-option

Q-77 ⇒ a-option

Q-78 ⇒ b-option

Q-79 ⇒ a-option

Q-80 ⇒ d-option.

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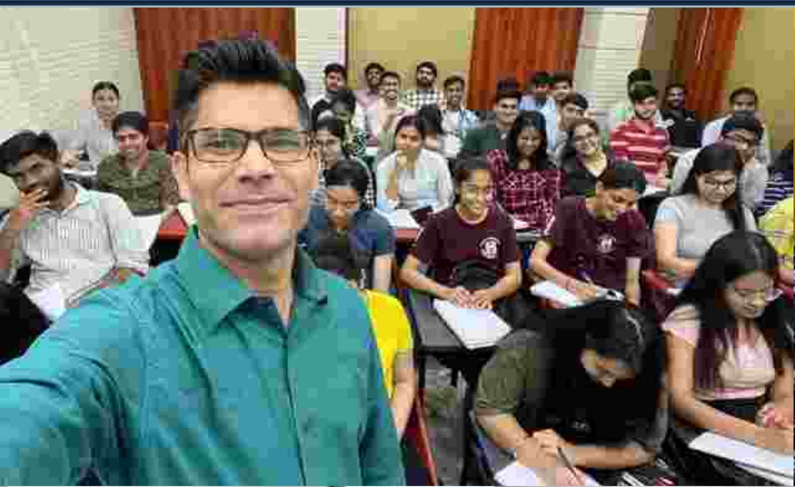


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