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Statistical Quality Control (S.Q.C) ⇒

S.Q.C is one of the most Important applications of the statistical techniques in Industry.

These techniques are based on the theory of probability and Sampling.

Quality ⇒ By quality we mean an attribute of the product that determines its fitness for use.

Quality means level / standard of the product, depends on four factors.

(i) Quality of Materials.
(ii) Quality of Manpower.
(iii) Quality of Machines.
(iv) Quality of Management.

Basis (आधार) of S.Q.C ⇒

The basis of S.Q.C is the degree of Variability in the size or the magnitude of a given characteristic (quality) of the product.

These variations in quality are classified as being due to Two Causes.

(i) chance causes

(ii) assignable causes.

Chance Causes ⇒ Some stable pattern of variation or a constant cause system is inherent (जन्मजात) in Any particular scheme of production and inspection.

This pattern results from many minor causes that behave in a Random manner.

The Variation due to these causes is beyond the control of Human hand and can not be prevented (रोकना) or eliminated (खतम करना) under any circumstances.

one has got to allow for Variation within this stable pattern, usually termed as allowable Variation. "The Range of Such Variation is Known as Natural Tolerance of the process."

Assignable Causes ⇒ The Second Type of Variation attributed to any production process is due to non-random or assignable causes and is termed as preventable Variation (रोकने योग्य). The assignable causes may occure in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods. Some of the important factors of assignable causes of Variation are Defective raw materials, New Techniques or operations, negligence (लापरवाही) of the operators, Improper Handling of machines, faulty Equipment, Unskilled Technical staff and so on.

These causes can be identified and eliminated before the production becomes defective.

NOTE- A production process is said to be in a State of statistical control, if it is governed by chance causes alone, in the absence of assignable causes of Variation.

Difference Between chanse and Assignable Cause of Variation ⇒

Chance Cause of Variation	Assignable Causes of Variation
(i) Consists of many Individual Causes	Consists of just a few Individual Causes
(ii) Any one chance cause results in only a Small amout of Variation	Any one assignable cause can result in a Large ammount of Variation
(iii) chance Variation can not be eleminated from the process	Assignable Variation can be eliminated from the process
(IV) Some typical chance Causes are (i) Slight Vibration of a machine. (ii) Lack of Human perfection in reading Instruments. (iii) Voltage fluctuations and Variation in Temperatures.	Some Typical assignable causes of Variation are. (i) Negligence of operators (ii) Defective raw materials (iii) Faulty Equipment (IV) Improper Handling of machines.

Definition and Benefits of S.Q.C ⇒

S.Q.C refers to the systematic control of those variables encountered in a manufacturing process which affect the excellence of the end product. Such variables are from the application of materials, men, machines and manufacturing conditions.

S.Q.C also enables us to decide whether to Reject or accept a particular product.

Benefits ⇒

(i) An obvious advantage of S.Q.C is the Control, maintenance and Improvement in the quality standards.

(ii) It Tells us when to leave a process alone and when to make action to correct troubles.

(iii) It provides better quality assurance at Lower inspection cost.

(iv) S.Q.C reduce waste of Time and material to the absolute minimum by giving an early warning about the occurrence of defects.

Process control and Product control ⇒

In Process control the proportion of defective items in the production process is to be minimized and it is achieved through the technique of control charts. i.e we want to ensure that the proportion of defective items in the manufactured product is NOT too Large.

Product control means that controlling the quality of the product by critical examination through Sampling. i.e it ensure that the product marketed by Sale department does not contain a Large no of defective items.

Thus product control is concerned with classification of Semi-finished goods or finished goods into acceptable or Rejectable item.

NOTE :
process control → defective items कम बनने चाहिये
Product " → " " बिकने "

Control limits, Specification limits, Tolerance limits ⇒

(i) Control limits ⇒ These are limits of Sampling Variation of a statistical measure (e.g mean, range, or fraction-defective) such that if the Production process is under control, the values of the measure (like mean) calculated from different samples will lie within these limits.

Points falling outside control limits indicate that the process is NOT operating under a system of chance causes i.e assignable causes of variation are present, which must be eliminated.

(ii) Specification limits ⇒ (limits for quality) ⇒

When an article is proposed to be manufactured, the manufacturers have to decide upon the maximum and the minimum allowable dimensions of some quality characteristic so that the product can be gainfully utilised for which it is intended.

If the dimensions are beyond these limits, the product is treated as defective and can-not be used.

These maximum and minimum limits of variation of individual items, as mentioned in the product design, are known as specification limits.

(iii) Tolerance limits ⇒

these are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie with a given prob^{t}, provided the process is in a state of statistical quality control.

Ex:- we may claim with a prob^{t} of 0.99 that at least 90% of the product will have dimensions between some stated limits l_1 and l_2. These limits l_1 and l_2 are known as statistical tolerance limits.

Control charts ⇒ Define Control charts ⇒

Control chart is a simple pictorial device for detecting Unnatural patterns of Variations in data resulting from Repetitive processes. i.e control charts provide criteria for detecting Lack of statistical control.

Control charts tell us at a glance whether the sample point falls within 3σ control limits or not. Any sample point going outside the 3σ limits is an Indication of the Lack of statistical control i.e Presence of some assignable cause of Variation which must be traced, Identified, and eliminated.

A Typical control chart consists of the following 3 Horizontal lines.

(i) A Central line (C.L), indicating the desired standard or the level of the process.

(ii) upper Control Limit (U.C.L), indicating the upper limit of Tolerance.

(iii) Lower Control limit (L.C.L), indicates the Lower limit of Tolerance.

Major parts of a Control chart ⇒ A C.C includes the following four major parts.

(1) Quality Scale ⇒ this is a Vertical Scale. The scale is marked according to the quality characteristics (either in Variables or in attributes) of each Sample.

(2) Plotted Samples ⇒ The quality of individual items of a sample are not shown on a control chart. only the quality of the entire Sample represented by a Single Value (of a Statistics) is plotted. The Single Value plotted on the chart is in the form of a Dot.

(3) Sample numbers ⇒ The Samples plotted on a Control chart are numbered Individually and Consecutively on a Horizontal line. Generally 25 Samples are used in constructing a control chart.

(4) The Horizontal lines ⇒ Here we have 3 Horizontal lines.

The central line represents the average quality of the samples plotted on the chart. i.e $C.L = E(t)$

The line above the C.L shows the upper control limit U.C.L which is obtained by adding 3σ to the average i.e $U.C.L = E(t) + 3\,S.E(t)$.

The line below the central line is the Lower Control Limit L.C.L. which is obtained as $L.C.L = E(t) - 3 \cdot S.E(t)$.

Tools (Techniques) for S.Q.C ⇒

The following four techniques, are the most Important statistical tools for data analysis in S.Q.C.

(1) Control chart for Variables ⇒ This is used for a characteristic which can be measured quantitatively. ie diameter of screw, life of an electric bulb. etc. Such Variables are of continuous type and are regarded as follow normal prob^t Law. For quality control of such data, Two types of control charts are used.

(a) charts for $\bar{X}$ (mean) and R (Range)

(b) charts for $\bar{X}$ (mean) and σ (standard deviation)

(2) Control chart for fraction defective or p-chart ⇒ This chart is used if we are dealing with attributes in which case the quality characteristics of the product are NOT measurable quantitively but can be identified by their absence or presence from the product

or by classifying the product as defective or non-defective.

(3) Control chart for the Number of defects per Unit or (c-chart) ⇒ This is usually used with advantage when the characteristic representing the quality of a product is a discrete Variable. i.e the number of defective rivets (कील) in an aircraft wing.

(4) The portion of the Sampling theory which deals with the quality Protection given by any specified Sampling acceptance procedure (this is for product Control).

Control charts for Variables ($\bar{x}$ and R charts) ⇒

These charts may be applied to any quality characteristic that is measurable. In order to control a measurable characteristic we have to exercise control on the measure of location as well as the measure of dispersion.

Usually $\bar{x}$ and R charts are employed to control the location and dispersion respectively of the characteristic.

Steps for $\bar{x}$ and R charts ⇒

(i) Correct Measurement ⇒ Since the conclusions drawn from control chart are broadly based on the variability in the measurements as well as the variability in the quality being measured, it is important that the mistakes in Reading measurement instruments or errors in Recording data should be minimised so as to draw valid conclusions from control charts.

(ii) Selection of Samples ⇒ Usually the Sample Size n is taken to be 4 or 5 while the frequency of Sampling depends on the state of the control exercised. Initially more frequent Samples will be required and once a state of control is maintained, the frequency may be relaxed.

Normally 25 samples of size 4 each or 20 Samples of size 5 each under control will give good estimate of the process Average and dispersion.

(3) Calculation of $\bar{X}$ and R for each Sample ⇒

let X_{iJ} ; $J = 1, 2, \cdots n$ be the measurements on the i^{th} sample, $i = 1, 2, \cdots K$.

The mean $\bar{X}_i$, the Range R_i and standard deviation s_i for the i^{th} sample are given by

$$\bar{X}_i = \frac{1}{n}\sum_{J=1}^{n} X_{iJ} \quad ; \quad R_i = \underset{J}{Max}\{X_{iJ}\} - \underset{J}{Min}\{X_{iJ}\}$$

$$s_i^2 = \frac{1}{n}\sum_{J}(X_{iJ} - \bar{X}_i)^2 \quad \forall \; i = 1, 2, \cdots K.$$

next we find $\bar{\bar{X}} = \frac{1}{K}\sum_{i=1}^{K}\bar{X}_i$; $\bar{R} = \frac{1}{K}\sum_{i=1}^{K} R_i$; $\bar{S} = \sum_{i=1}^{K} s_i$

(4) Setting of Control limits $\Rightarrow$

If σ, standard deviation of pop^n is Known then

$S.E(\bar{X}_i) = \sigma/\sqrt{n} \quad \forall \; i = 1, 2, \cdots k.$

Also from the Sampling distribution of Range we Know $E(\bar{R}) = d_2 \cdot \sigma$

where d_2 is Constant depends on n.

$\Rightarrow \bar{R} = d_2 \cdot \hat{\sigma} \Rightarrow \hat{\sigma} = \bar{R}/d_2 \rightarrow$ if σ is not known

also $\bar{\bar{X}}$ is an U.E of pop^n mean μ i.e

$E(\bar{\bar{X}}) = \mu \Rightarrow \hat{\mu} = \bar{\bar{X}}$

Control limits for $\bar{X}$-chart $\Rightarrow$

Case-I $\Rightarrow$ If both μ and σ are Known.

The 3-σ Control limits for $\bar{X}$ chart are

$E(\bar{X}) \pm 3 \cdot S.E(\bar{X})$

$\Rightarrow \mu \pm 3 \cdot \sigma/\sqrt{n} \Rightarrow \mu \pm A \cdot \sigma \quad \{ A = 3/\sqrt{n}$

$\Rightarrow$ C.L $= \mu$

U.C.L $= \mu + A \cdot \sigma$

L.C.L $= \mu - A \cdot \sigma$

where $A = 3/\sqrt{n}$ is calculated by the given table for different Values of n.

Case-II. If μ and σ are not known.

we use estimates for μ and σ as

$$\hat{\mu} = \bar{\bar{X}} \quad \text{and} \quad \hat{\sigma} = \frac{\bar{R}}{d_2}$$

Hence the 3-σ limits are.

$$C.L \equiv \bar{\bar{X}}$$

$$U.C.L \equiv \bar{\bar{X}} + 3 \cdot \bar{R}/d_2\sqrt{n} = \bar{\bar{X}} + A_2 \cdot \bar{R} \qquad \left\{ A_2 = \frac{3}{d_2\sqrt{n}} \right.$$

$$L.C.L \equiv \bar{\bar{X}} - 3 \cdot \bar{R}/d_2\sqrt{n} = \bar{\bar{X}} - A_2 \bar{R}$$

where A_2 depends on n and its values are calculated from table for different n.

NOTE- If, on the other hand, the control limits are to be obtained in terms of $\bar{s}$ rather than $\bar{R}$. then $E(\bar{s}) = c_2 \cdot \sigma$

c_2 is constant depend on n.

$$\Rightarrow \hat{\sigma} = \frac{\bar{s}}{c_2}$$

$$\Rightarrow \text{limits are} \quad \bar{\bar{X}} \pm \frac{3 \cdot \bar{s}}{\sqrt{n} \cdot c_2} \Rightarrow \bar{\bar{X}} \pm A_1 \bar{s}$$

where $A_1 = \frac{3}{\sqrt{n} \cdot c_2}$ is calculated from table for different values of n.

Control limits for R-chart ⇒

R-chart is constructed for controlling the Variation in the dispersion (Variability) of the product.

We have the following steps.

(i) find $R_i = \max_j X_{ij} - \min_j X_{ij} \quad \forall\ i = 1, 2, \dots K$

(2) find $\bar{R} = \frac{1}{K}\sum_{i=1}^{K} R_i$

(3) The 3-σ Control limits for R-chart are

$$E(R) \pm 3\cdot\sigma_R$$

where $E(R)$ is estimated by $\bar{R}$. and σ_R is estimated as $\hat{\sigma}_R = d_3\hat{\sigma} = d_3\cdot\frac{\bar{R}}{d_2}$; if σ is Unknown.

$$\Rightarrow \quad U.C.L_R = E(R) + 3\cdot\hat{\sigma}_R = \bar{R} + 3\frac{d_3}{d_2}\cdot\bar{R} = D_4\bar{R} \quad ; \quad D_4 = 1 + 3\frac{d_3}{d_2}$$

$$L.C.L_R = E(R) - 3\cdot\sigma_R = \bar{R} - 3\frac{d_3}{d_2}\bar{R} = D_3\bar{R} \quad ; \quad D_3 = 1 - 3\frac{d_3}{d_2}.$$

$$C.L_R = E(R) = \bar{R}$$

The values of D_3 and D_4 are from Table

if σ is Known:- then $E(R) = d_2\cdot\sigma$

and $\sigma_R = d_3\cdot\sigma$

$$\Rightarrow \quad U.C.L_R = E(R) + 3\sigma_R = d_2\cdot\sigma + 3d_3\sigma = D_2\sigma \quad ; \quad D_2 = d_2 + 3d_3$$

$$L.C.L_R = E(R) - 3\sigma_R = D_1\sigma \quad ; \quad D_1 = d_2 - 3d_3$$

$$C.L_R = E(R) = \bar{R}$$

NOTE- Since Range can never be negative, $L.C.L_R$ must be positive or zero. If it comes out to be negative, it is taken as 0.

How To Construct control charts for $\bar{X}$ and $\bar{R}$ ⇒

plotting of Central line and control limits. control charts are plotted on a rectangular co-ordinate axis.

Vertical scale representing the statistical measure $\bar{X}$ and R, and Horizontal scale represents the sample number.

Sample points (mean or Range) are indicated on the chart by points.

For $\bar{X}$-chart, the central line is drawn as a solid Horizontal line at $\bar{\bar{X}}$ and $U.C.L_{\bar{X}}$ and $L.C.L_{\bar{X}}$ are drawn at the computed values as dotted Horizontal lines.

Similarly for $\bar{R}$-chart.

*Note ⇒ $\bar{X}$-chart reveals (पता करना) undesirable variations between samples as far as their Averages are concerned. while the R-chart reveals Any undesirable variation within samples.

Repetition of Control limits (Modified Control limits for Future Use) ⇒

If all the points in both the charts remain within trial control limits, then these limits are accepted as final, and used for maintaining control charts for subsequent production.

If, some of the points go outside the limits in one of the charts then it is concluded that these samples were produced when the process was not in control. and these samples are Rejected.

Then a Second Set of Trial limits is Constructed, using only the remaining Samples, and using these fresh control limits, new charts are constructed and the remaining samples are plotted on the new chart.

We Repeat this process untill all sample points lies within Limits in both charts.

These final limits are modified control limits for

Future use

Criterion for Detecting Lack of Control in $\bar{X}$ and R-charts ⇒

The pattern of the sample points in a control chart is also the criterion for detecting lack of control.

The following situations depict lack of control.

(i) A point outside the control limits ⇒

A point going outside control limits is a clear indication of the presence of assignable causes of variation which must be searched and corrected.

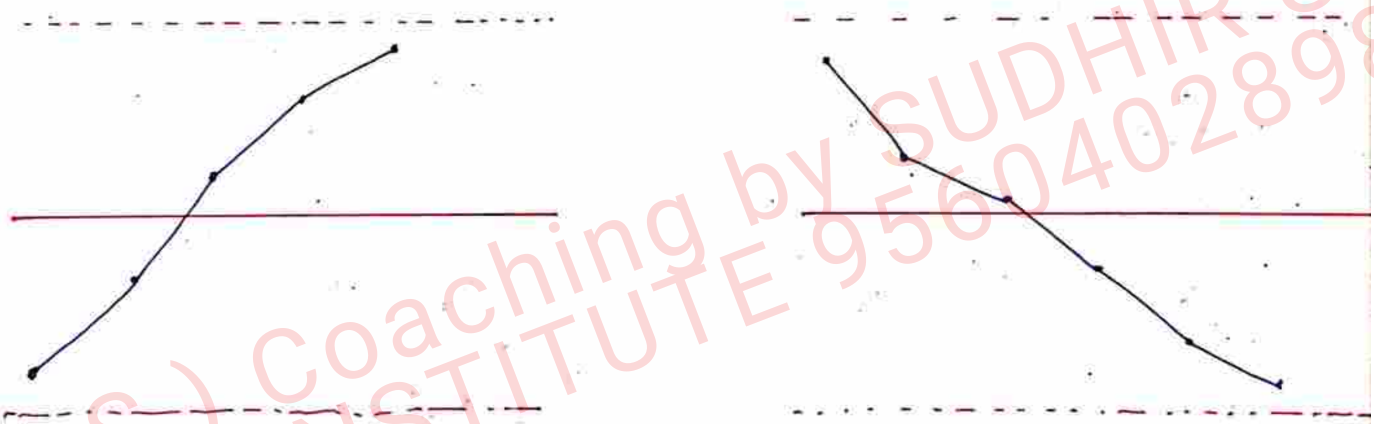
(ii) A Run of Seven or more points ⇒

Although all the sample points are within control limits, usually the pattern of points (sample points) in the chart indicates assignable causes. One such situation is a run of 7 or more sample points above or below the central line in the control chart.

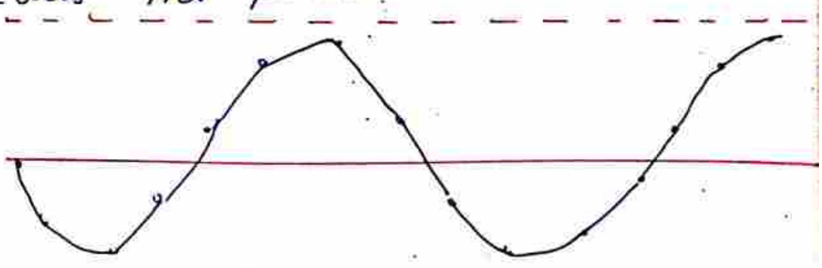
(3) a Run of 2, 3 points beyond $2\text{-}\sigma$ limits or a Run of 4, 5 sample points beyond $1\text{-}\sigma$ limit indicates assignable causes.

(4) The sample points on $\bar{X}$ and R-charts, too close to the central line, exhibit (प्रदर्शित) another form of assignable cause. thus shows biases in measurements.

(5) Presence of Trend ⇒ The Trends exhibited by sample points on the control charts are also an Indication of assignable cause.



(6) Presence of cycles ⇒ In some cases the cyclic pattern of points in the control chart indicates the presence of assinable cause of variation



Interpretation of $\bar{x}$ and R-charts ⇒
(व्याख्या)

The process should be deemed in statistical control if both the charts show a state of control. but if charts are not in control, we summarise ~~of~~ in a tabular form, such different situations and the interpretation to be accorded to each.

No	Situation In		Interpretation (व्याख्या)
	R-chart	$\bar{x}$-chart	
1	In Control	Points beyond limits only on one-side	level of Process has shifted.
2	In Control	Points beyond limits on both-side	level of Process is changed
3	out of Control	Points beyond limits on both side	Variability has Increased
4	out of Control	out of control on one-side	both level and Variability have changed
5	In Control	Run of 7 or more points on one side of central line	Shift in Process level
6	In Control	Trend of 7 or more Points. no points outside Control limits	Process level is gradually (धीरे-धीरे) changing
7	Runs of 7 or more points above Control line	—	Variability has Increased
8	Points Too closed to the central line	—	bias in Measurement
9	—	Points too closed to the central line	bias in Measurement

Control chart for Standard deviation (σ-chart) $\Rightarrow$

If the Sample size n is large, then standard deviation is an ideal measure of dispersion, so a combination of control chart for mean $\bar{x}$ and standard deviation (s), known as $\bar{x}$ and s-chart or ($\bar{x}$ and σ-chart) is theoretically more appropriate than a combination of $\bar{x}$ and R-chart for controlling process average and process Variability.

In a random sample of size n from normal pop^n with standard deviation σ, we have.

$$E(s^2) = \frac{n-1}{n}\cdot\sigma^2$$

$$\Rightarrow \quad E(s) = c_2\cdot\sigma \quad \text{where} \quad c_2 = \sqrt{\frac{2}{\pi}}\cdot\frac{[(n-2)/2]!}{[(n-3)/2]!}$$

$$\Rightarrow \quad V(s) = E(s^2) - (E(s))^2$$

$$= \left(\frac{n-1}{n} - c_2^2\right)\sigma^2$$

$$\Rightarrow \quad S.E(s) = c_3\cdot\sigma \quad \text{where} \quad c_3 = \sqrt{\left(\frac{n-1}{n} - c_2^2\right)}$$

$$\Rightarrow \quad U.C.L_s = E(s) + 3.S.E(s) = (c_2 + 3c_3)\cdot\sigma = B_2\cdot\sigma$$

$$L.C.L_s = E(s) - 3.S.E(s) = (c_2 - 3c_3)\cdot\sigma = B_1\,\sigma$$

$$C.L_s = E(s) = c_2\cdot\sigma$$

The values of B_1 and B_2 have been tabulated for different values of n.

NOTE:- If the value of σ is not known, then

$$\hat{\sigma} = \frac{\bar{S}}{C_2} \qquad \left\{ \begin{array}{l} E(S) = C_2 \cdot \sigma \\ \Rightarrow E(\bar{S}) = C_2 \cdot \sigma \end{array} \right.$$

$$\Rightarrow U.C.L_S = E(S) + 3 \cdot S.E(S) = \bar{S} + 3 \cdot \frac{C_3}{C_2} \cdot \bar{S} = B_4 \cdot \bar{S}$$

$$L.C.L_S = E(S) - 3 \cdot S.E(S) = \bar{S} - 3 \cdot \frac{C_3}{C_2} \cdot \bar{S} = B_3 \bar{S}$$

$$C.L_S = \bar{S}$$

Control chart for attributes ⇒

$\bar{X}$ and R (or σ) charts are used for Variables only i.e for the quality characteristic which can be measured and expressed in numbers. So In case for attributes i.e in case where quality characteristic is observed only as attributes by classifying an item as defective or non-defective.

There are Two control charts for attributes.

(i) Control chart for fraction defective (p-chart) or the number of defectives (np or d chart)

(ii) Control chart for the number of defects per Unit (c-chart)

Control chart for fraction Defective (p-chart) $\Rightarrow$

While dealing with attributes, a process will be adjudged (निर्णीत) in statistical control if all the samples are ascertained (निर्धारित) to have the same pop^n proportion P.

If d is the number of defectives in a sample of size n, then the sample proportion defective is $p = d/n$.

$$\Rightarrow \quad d \sim b(n, P)$$

$$\Rightarrow \quad E(d) = nP \quad ; \quad V(d) = nPQ \quad ; \quad Q = 1-P$$

$$\Rightarrow \quad E(p) = E(d/n) = \frac{1}{n} E(d) = P$$

$$V(p) = \frac{1}{n^2} V(d) = \frac{PQ}{n}$$

$\Rightarrow$ 3σ control limits for p-chart are given by

$$E(p) \pm 3\, S.E(p) = P \pm 3\sqrt{\frac{PQ}{n}} = P \pm A \cdot \sqrt{PQ}$$

where $A = 3/\sqrt{n}$; provided P is known

If P is NOT known :-

Let d_i be the number of defectives and p_i the fraction defective for the i^{th} sample ($i = 1, 2, \dots k$) of size n_i.

Then the pop^n proportion P is estimated by the statistics $\bar{p}$ given by

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

Now

$$E(\bar{p}) = \frac{\sum E(d_i)}{\sum n_i} = \frac{\sum n_i P}{\sum n_i} = P$$

$\Rightarrow$ $\bar{p}$ is U.E of P.

$\Rightarrow$ $U.C.L_p = \bar{p} + A\sqrt{\bar{p}\bar{q}}$ $\qquad \bar{q} = 1-\bar{p}$

$L.C.L_p = \bar{p} - A\sqrt{\bar{p}\bar{q}}$

$C.L_p = \bar{p}$

Control chart for Number of defectives (d-chart)

If Instead of p, the Sample proportion defective, we use d, the number of defectives in the Sample, then the 3-σ control limits for d-chart are given by

$$E(d) \pm 3 \cdot S \cdot E(d)$$

$$\Rightarrow \quad nP \pm 3\sqrt{nPQ} \quad ; \quad P \text{ is Known.}$$

If P is NoT Known. then 3-σ limits are

$$n\bar{p} \pm 3\sqrt{n\bar{p}\,\bar{q}}\,.$$

Control chart for number of defects per Unit (c-chart) ⇒

first we discuss differences between defect and defective. An article which does not conform to one or more of the specification, is termed as defective while any instance of article's Lack of Conformity to specification is a defect. Thus, every defective contains one or more of the defects.

c-chart applies to the number of defects per Unit. Sample size for c-chart may be 1 Unit like a Radio

Control limits for c-chart ⇒ In many Inspection situations, the Sample size n is very large. and the prob^t p of the occurrence of a defect in any one spot is very small s.t $n \cdot p$ is finite.

In such situations from statistical theory we know that the pattern of Variations in data can be represented by poisson distribution and Consequently 3-σ control limits based on poisson distribution are used.

If we assume that $c \sim P(\lambda)$

$\Rightarrow \quad E(c) = \lambda$ and $V(c) = \lambda$

$\Rightarrow$ 3-σ control limits for c-chart are given by.

$$U.C.L_c = E(c) + 3\sqrt{V(c)} = \lambda + 3\sqrt{\lambda}$$

$$L.C.L_c = E(c) - 3\sqrt{V(c)} = \lambda - 3\sqrt{\lambda}$$

$$C.L_c = \lambda$$

provided λ is Known.

If λ is not Known $\Rightarrow$ If the value of λ is not Known, it is estimated by the mean number of defects per unit.

Thus, If c_i is the number of defects observed on the i^{th} Inspected Unit; $i = 1, 2, \cdots k$.

then an estimate of λ is given by

$$\hat{\lambda} = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$$

$$\Rightarrow \quad U.C.L_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$L.C.L_c = \bar{c} - 3\sqrt{\bar{c}}$$

$$C.L_c = \bar{c}$$

U-chart (c-chart for Variable Sample Size) ⇒

If this case instead of plotting c, the statistics $u = c/n$ is plotted, n being the sample size which is varying.

If n_i is the sample size and c_i the Total number of defects observed in the i^{th} sample then $u_i = c_i/n_i$; $i = 1, 2, \dots k$.
gives the average number of defects per unit for the i^{th} sample.

In this case an estimate of λ, the mean number of defects per unit in the lot, based on all the k-samples is given by

$$\hat{\lambda} = \bar{u} = \frac{1}{k}\sum_{i=1}^{k} u_i$$

$$\Rightarrow E(\bar{u}) = \lambda$$

$$\Rightarrow \sqrt{V(\bar{u})} = \sqrt{\lambda}/\sqrt{n} = S \cdot E(\bar{u})$$

$$\left\{ V(\bar{x}) = \sigma^2/n \right.$$

$$\hat{\lambda} = \bar{u}$$

$$U.C.L_u = \bar{u} + 3\sqrt{\bar{u}/n}$$

$$L.C.L_u = \bar{u} - 3\sqrt{\bar{u}/n}$$

$$C.L_u = \bar{u}$$

Natural Tolerance limits and specification limits ⇒

If μ and σ are the process average and process standard deviation respectively, then the limits $\mu \pm 3\sigma$ are called the Natural Tolerance limits (which are different from the control limits i.e $\mu \pm 3\sigma/\sqrt{n}$).

The prob^t of an observation lying outside these limits is 0.0027.

The width 6σ which is the Inherent (जन्मजात) Variability of the process is given a special name Natural Tolerance. this is maximum Acceptance Variation.

If μ and σ are not known then $\hat{\mu} \pm 3\hat{\sigma}$ are the estimates of the natural Tolerance limits where $\hat{\mu} = \bar{\bar{x}}$ and $\hat{\sigma} = \bar{R}/d_2$ or $\bar{s}/c_2$.

It might Happen that even though the process is in statistical control as exhibited by control charts, the customer may not be satisfied with the product.

This happens when the process does not conform to specification limits (Limits fixed by the customer) for that item.

A decision, whether a process needs adjustment or not, can be made at the point by comparing Natural Tolerance limits and Specification limits.

If X_{max} and X_{min} denote the upper specification limit (U.S.L) and Lower specification limit (L.S.L) respectively for some quality characteristic. When both these limits are specified, a comparison of these with the Natural Tolerance limits may result in one of the three situations.

(a) Natural tolerance is considerably smaller than specified tolerance i.e $X_{max} - X_{min} > 6\sigma$

$\Rightarrow$ In such a case almost all the manufactured items will conform to specifications as long as the process is in statistical control.

(b) Specification limits coincide with Tolerance limits i.e $X_{max} - X_{min} \doteqdot 6\sigma$

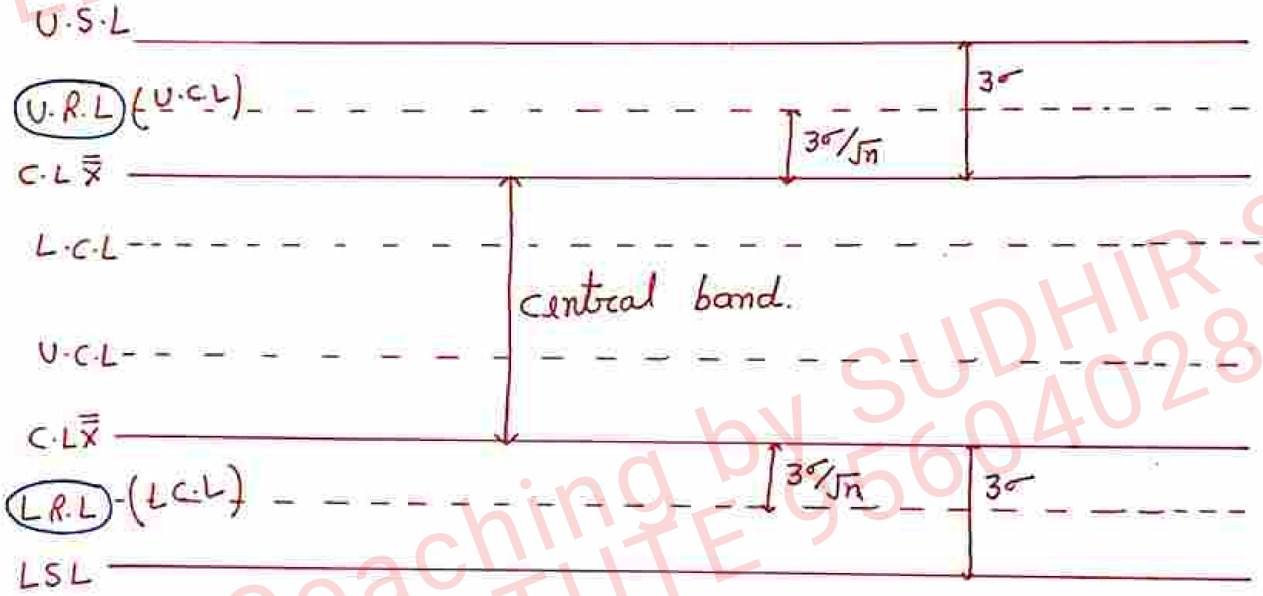
$\Rightarrow$ This is an Ideal situation and in this case a process in statistical control obviously implies that the product is meeting the specifications.

(c) Natural Tolerance is greater than specified tolerance i.e $X_{max} - X_{min} < 6\sigma$

$\Rightarrow$ If $X_{max} - X_{min} < 6\sigma$, then even with the process in control and the process average perfectly centered at the specification mean, the production of an appreciable quantity of defective articles (i.e articles not conforming to specifications) is Inevitable (अनिवार्य)

Modified control limits ⇒

If the specification limits lie outside the natural tolerances limits i.e $X_{max} - X_{min} > 6\sigma$, then Modified control limits which exhibit (प्रदर्शित) the relationship between the specification limits and the $\bar{x}$ values in $\bar{x}$-chart, may be used to permit shifts in process level within permissible limits.



The natural tolerances (natural dispersion) is 6σ. If the Universe is at the Highest accepting position i.e Upper specification limit (U.S.L), then the process average (central line) will be at a distance 3σ below U.S.L.

and Similarly when the universe is at its lowest acceptance position i.e. Lower specification limit (L.S.L). the process average is at a distance 3σ above the L.S.L.

Thus in this case, instead of fixed C.L at $\bar{\bar{x}}$, we have a central band (central area) so that as long as $\bar{x}$ lies in this central band, the product will conform to specifications.

So the upper and lower edges (boundaries) of the central band are given by

$$U.S.L - 3\sigma \quad , \quad L.S.L + 3\sigma$$

So for a sample of size n, as is clear from the figure, the highest and lowest satisfactory values of U.C.L and L.C.L, known as upper Rejection limit (U.R.L) and Lower Rejection limit (L.R.L) respectively are given by

$$U.R.L_{\bar{x}} = U.S.L - 3\sigma + 3\sigma/\sqrt{n}$$

$$L.R.L_{\bar{x}} = L.S.L + 3\sigma - 3\sigma/\sqrt{n}$$

These Rejection limits, when used in place of control limits, are called, Modified Control Limits.

Acceptance Sampling Inspection plans ⇒

Acceptance Sampling plans refer to the use of Sampling inspection by the purchaser (consumer) to decide whether to accept or to reject a lot of given product.

In statistical quality control terminology, it is also known as Product Control.

The Acceptance Sampling Inspection plan prescribes a procedure, that if applied to a Series of Lots, yields quality assurance by involving a decision to accept or Reject a Lot on the basis of Random Sampling drawn from it (Lot).

Acceptable Quality Level (AQL) ⇒

This is the quality level of a good lot.
It is the percent defective that can be
Considered satisfactory as a process average.
and represents a level of quality which
the producer wants accepted with a high
probability of acceptance. i.e
If α is the producer's Risk, then the level
of quality which results in $100(1-\alpha)\%$
acceptance of the good Lots submitted for
Inspection is called the Acceptable quality
level.
A Lot with relatively small fraction defective
(i.e Sufficiently good quality) say, p_1 that
we do not wish to reject more often
than a small proportion of time (कम समय में)
is sometimes referred to as a good Lot.

Usually

$P[\text{Rejecting a Lot of quality } p_1] = 0.05 = (\alpha)$.

$P_\alpha = P[\text{Accepting of a Lot of quality } p_1] = 0.95 = 1-\alpha$

then p_1 is Known as the A.Q.L and a lot of thus quality is considered as satisfactory by the consumer.

Lot Tolerance Percentage Defective (LTPD) ⇒

The lot tolerance proportion (percentage) defective denoted by p_t, is the Lot quality which is considered to be bad by the consumer.

The consumer is not willing to accept Lots having proportion defective p_t or greater.

$100 p_t$ is called L.T.P.D. i.e

this is the quality Level which the consumer regards as rejectable and is usually Read as R.Q.L (Rejecting Quality Level).

Process Average Fraction Defective ($\bar{p}$) ⇒

The process average fraction defective $\bar{p}$ of any manufactured product is obtained by finding the percentage of defective in the product over a fairly long time.

Consumer's Risk ⇒ Any sampling scheme would involve certain risk on the part of the consumer, in the sense that he has to accept lots of quality p_t or greater fraction defective.

Consumer's Risk $= P_c = \beta = P[\text{Accept a lot of quality } p_t]$

Producer's Risk ⇒ The producer has also to face the situation that some good lots will be rejected.

The prob^t of rejecting a lot with $100\bar{p}$ as the process average percentage defective is called the producer's risk. P_p and is usually denoted by α.

⇒ Producer's Risk $= P_p = \alpha = P$[Rejecting a lot of quality $\bar{p}$]

NOTE. $\bar{p}$ is the quality level, process average fraction defective, which is satisfactory level.

Rectifying Inspection plans ⇒
(सुधार)

In the following sections we shall discuss lot by lot sampling plans in which a specified quality objective is attained through corrective inspection of rejected lots.

The inspection of the rejected lots and replacing the defective pieces found in the rejected lots by the good ones, eliminates the number of defectives in the lot to a great extent, thus improving the lot quality. These plans are called Rectifying Inspection plans.

The two Important points related to rectifying inspection plans are.

(i) The average quality of the product after sampling, and 100% inspection of Rejected lots. called Average out going Quality (A.O.Q).

(ii) The average amount of inspection required for the rectifying inspection plan, called Average total Inspection (A.T.I).

Average outgoing Quality limit (AOQL) ⇒

let the producer's fraction defective i.e. Lot quality before inspection be p. this is Termed as Incoming quality. The fraction defective of the Lot after Inspection is known as out going quality of the Lot.

The expected fraction defective remaining in the Lot after the application of the Sampling Inspection plans is Termed as Average outgoing quality (A.O.Q) $\tilde{p}$.

it is the f^n of Incoming quality p. and the AOQ Values are given by the formula

$$\tilde{p} = AOQ = \frac{p(N-n) \cdot P_a}{N} \quad \text{———} \; *$$

where N is lot size, n is Sample size from Lot. P_a is the prob^t of acceptance of the Lot.

the formula * assumes that all defectives found are repaired or replaced by good pieces.

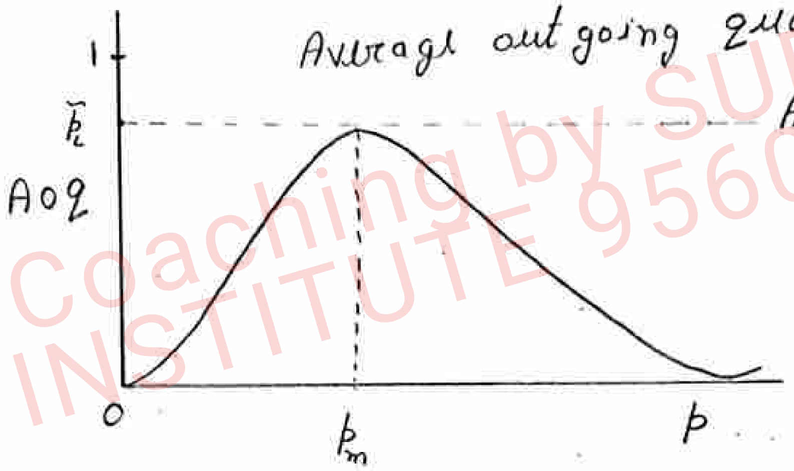
In general, if p is the incoming quality and a rectifying inspection plan calling for 100% inspection of the rejected lots is used, then the AOQ of the Lot will be given by

$$\hat{p} = AOQ = p \cdot P_a(p) + 0 \cdot [1 - P_a(p)] = p \cdot P_a(p) \quad \text{——} \; {*}{*}$$

because (i) $P_a(p)$ is the prob^t of accepting the Lot of quality p and when the Lot is accepted on the basis of the inspection plan, the outgoing quality of the Lot will be approximately same as the incoming lot quality p. and

(ii) $1 - P_a(p)$ is the prob^t of rejection of the Lot and when the Lot is rejected after Sampling inspection and is Subjected to 100% Screening and rectification, the AOQ is zero.

For a given Sampling plan, the Value of AOQ can be plotted for different Values of p to obtain the AOQ curve as given in Fig.



From $**$, we find that if $p=0$ i.e. Lot is 100% O.K. then $AOQ = 0$ and if $p=1$ i.e. Lot is 100% defective then $P_a(p) = 0$ and so $AOQ = 0$.

For other Values of p lying between 0 and 1, the AOQ will be positive and will have a maximum Value for some Value of the incoming quality $p = p_m$

The maximum Value of AOQ $\tilde{p}$ subject to Variation in p is called the Average outgoing Quality Limit ($\tilde{p}_L$).

If p_m is the value of p which maximises $\tilde{p}$ (AOQ) in * then

$$\tilde{p}_L = AOQL = \frac{p_m (N-n) \cdot P_a}{N} \quad \text{--- (i)}$$

where P_a is to be computed for $p = p_m$.

we can write Eq^n (i) as

$$AOQL = \tilde{p}_L = \frac{y}{n}\left(1 - \frac{n}{N}\right) \quad \text{where } y = n p_m \cdot P_a$$

and y has been tabulated for various values of n.

O.C Curve ⇒ O.C (operating characteristic) Curve of a Sampling plan is a graphic representation of the relationship between the probability of acceptance $P_a(p)$ or denoted as $L(p)$, for variations in the incoming lot quality p.

Average Sample Number (ASN) and Average Amount of Total Inspection (ATI) ⇒

The average sample number (ASN) is the expected value of the sample size required for coming to a decision about the acceptance or rejection of the lot in an acceptance-rejection sampling plan. obviously it is a f^n of the incoming lot quality p.

on the other Hand, the Expected number of items inspected per lot to arrive at a decision in an Acceptance-Rectification Sampling inspection plan calling for 100% inspection of the Rejected Lots. is called Average amount of Total Inspection (A.T.I).

obviously A.T.I is also a f^n of the lot quality p.

we observe that

ATI = ASN + (Average size of Inspection of the Remainder in the Rejected Lots)

thus, if the lot is accepted on the basis of the sampling inspection plan then ATI = ASN.

otherwise ATI > ASN.

However, for an acceptance - Rectification single sampling plan calling for 100% inspection of the Rejected lots, additional $(N-n)$ items will have to be inspected for each rejected Lot.

Thus, in this case, the number of items inspected per lot varies from lot to lot and is equal to n if the lot is accepted and equal to N if the lot is rejected on the basis of the sampling inspection plan.

Hence the average amount of Total Inspection is a f^n of the lot quality p and is given by

$$ATI = n \cdot L(p) + N \cdot (1 - L(p)) \qquad \{ L(p) = P_a(p)$$

$$\Rightarrow \quad ATI = n L(p) + (N - n + n)[1 - L(p)]$$

$$= nL(p) + (N-n)[1-L(p)] + n[1-L(p)]$$

$$ATI = n + (N-n)[1-L(p)]$$

NOTE:- The actual sample size can not be fractional but the expected sample size may be obtained to the nearest decimal required

NOTE:- The ASN and ATI plotted against the lot quality p give the ASN curve and ATI curve respectively.

SAMPLING Inspection plans for attributes ⇒

The commonly used sampling inspection plans for attributes and count of defectives are.

(i) Single Sampling plan

(ii) Double " "

(iii) Sequential " "

(i) Single Sampling plan ⇒ If the decision about accepting or rejecting a lot is taken on the basis of one Sample only, the acceptance plan is described as Single Sampling plan. it is completely specified by three numbers N, n and c, where

N is the Lot size.

n is the Sample size.

c is the acceptance number i.e maximum allowable number of defectives in the Sample.

The Single Sampling plan may be decribed as follows

(i) Select a Random sample of size n from a Lot of size N.

(ii) Inspect all articles included in the sample. Let d be the number of defectives in the sample.

(iii) If $d \le c$, accept the Lot, replacing defective pieces found in the sample by non-defective ones.

(iv) If $d > c$, Reject the lot. In this case we inspect the entire Lot and replace all the defective pieces by good ones.

NOTE:- The basic problem in administering a Single Sampling plan is the choice of n and c.

Determination of n and c ⇒

The lot size N is fixed and Known. thus the Two unKnown quantities that need to be determined in the sampling plan are n and c.

In a lot of incoming quality p, the number of defective pieces is $N \cdot p$ i.e Total Size × fraction defective. and non-defective pieces is

$$N - Np = N(1-p)$$

So the prob^t of getting exactly x defectives in a sample of size n from this lot is given by Hyper-geometric distribution.

$$g(x, p) = {}^{Np}C_x \cdot {}^{N-Np}C_{n-x} / {}^{N}C_n$$

Prob^t of accepting a lot of quality p is

$$P_a(p) = \sum_{x=0}^{c} g(x, p) = \sum_{x=0}^{c} {}^{Np}C_x \cdot {}^{N-Np}C_{n-x} / {}^{N}C_n \quad \text{--- (i)}$$

Hence the Consumer's Risk is given by

$$P_c = P[\text{Accepting a lot of quality } p_t]$$

$$= \sum_{x=0}^{c} g(x, p_t) = \sum_{x=0}^{c} {}^{Np_t}C_x \cdot {}^{N-Np_t}C_{n-x} / {}^{N}C_n \quad \text{--- (ii)}$$

The producer's Risk is given by

$$P_p = P[\text{Rejecting a lot of quality } \bar{p}]$$

$$= 1 - \sum_{x=0}^{c} g(x, \bar{p}) = 1 - \sum_{x=0}^{c} {}^{N\bar{p}}C_x \cdot {}^{N-N\bar{p}}C_{n-x} / {}^{N}C_n \quad \text{--- (iii)}$$

If the process average fraction defective is $\bar{p}$ then the average amount of total Inspection per lot is

$$A.T.I = n + (N-n) P_p \quad \text{--- (4)}$$

because n items have to be inspected in each case and remaining $(N-n)$ items will be inspected only if $d > c$ i.e if the lot is rejected when the lot quality is $\bar{p}$. and the prob^t for this is P_p.

The Computation of Hyper-geometric probabilities in eq^n (ii) and (iii) is extremely difficult so we use binomial approximation to solve Eq^n (ii) and (iii).

$$P_c = \sum_{x=0}^{c} \frac{(Np_t)!}{x!\,(Np_t - x)!} \cdot \left(\frac{n}{N}\right)^x \left(1 - \frac{n}{N}\right)^{Np_t - x} \quad \text{— (5)}$$

$$P_p = 1 - \sum_{x=0}^{c} \frac{n!}{x!\,(n-x)!} \cdot (\bar{p})^x (1-\bar{p})^{n-x} \quad \text{— (6)}$$

[Reffer to book]

In most of the practical problems, $\bar{p}$ is small and n is likely to be large, Hence we use poisson approximation to binomial. Hence eq^n(6) can be written as

$$P_p = 1 - \sum_{x=0}^{c} \left[\frac{\lambda^x \cdot e^{-\lambda}}{x!}\right] \quad \text{where } \lambda = n\bar{p}$$

$$\Rightarrow A.T.I = n + (N-n)\left[1 - \sum_{x=0}^{c} \left\{\frac{e^{-n\bar{p}} \cdot (n\bar{p})^x}{x!}\right\}\right] \quad \text{— (7)}$$

Here Consumer's requirement fixes the Values of P_c and p_t. and N is always fixed.

For given values of P_c and p_t the eq^n (ii) which involves Two Unknowns n and c is satisfied by a large number of pairs of n and c.

To safeguard (रक्षा) producer's Interest also, out of these possible pairs one involving the minimum amount of inspection as given in Eq^n (4) is chosen.

AOQL ⇒ If p is the incoming quality, there will be no defective left in a lot of size N if the sample contains more than c defectives because in this case all products of the lot are rectifying. i.e if $x > c$.

on the other hand if $x \leq c$, the number of defectives in a lot of size N is $N \cdot p - x$.

thus the mean value of the number of defectives after sampling inspection is given by

$$m = \sum_{x=0}^{c} (N \cdot p - x) \cdot g(x, p) + \sum_{x=c+1}^{N} 0 \cdot g(x, p)$$

$$= \sum_{x=0}^{c} (Np - x) \cdot {}^{Np}C_x \cdot {}^{N-Np}C_{n-x} / {}^{N}C_n.$$

⇒ The mean values of fraction defective after inspection i.e AOQ will be

$$AOQ = \hat{p} = \frac{m}{N} = \sum_{x=0}^{c} \left(p - \frac{x}{N}\right) {}^{Np}C_x \cdot {}^{N-Np}C_{n-x} / {}^{N}C_n.$$

Subject to variation in p, $\check{p}$ has a maximum value, say, $\check{p}_L$ which is termed as AOQL.

O.C curve ⇒ The O.C curve for the incoming quality p is given by

$$P_a(p) = L(p) = \sum_{x=0}^{c} g(x, p)$$

$$= \sum_{x=0}^{c} {}^{Np}C_x \, {}^{N-Np}C_{n-x} \Big/ {}^{N}C_n \quad \text{---} (8)$$

If p small < 0.10, a good approximation to eq^n (8) is given as

$$L(p) \simeq \sum_{x=0}^{c} {}^{Np}C_x \left(\frac{n}{N}\right)^x \left(1 - \frac{n}{N}\right)^{Np-x}$$

also if n is Large.

$$L(p) \simeq \sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}$$

Double Sampling plan ⇒

Another Sampling scheme propounded by Dodge and Romig is the Second Sampling method (Double Sampling method) In this method, a Second Sample is permitted if the first Sample fails i.e if the data from the first sample is not conclusive on either side (about accepting or rejecting the Lot), then a definite decision is taken on the basis of the Second Sample. Such a rectifying double Sampling inspection plan for attributes is briefly described below.

$N \rightarrow$ Lot size.

$n_1 \rightarrow$ Size of Sample 1

$n_2 \rightarrow$ Size of " 2.

$c_1 \rightarrow$ maximum permissible number of defectives in first Sample if Lot is to be accepted without taking another Sample.

$c_2 \rightarrow$ Acceptance number for samples 1 and 2 Combined i.e maximum permissible number of defectives in combined samples if Lot is to be accepted

$d_1 \rightarrow$ number of defectives in Sample 1

$d_2 \rightarrow$ " " " " " 2.

procedure $\Rightarrow$

(i) take a Sample of size n_1 from the Lot of size N

(ii) If $d_1 \le c_1$, Accept the Lot, replacing the defectives found in the Sample by non-defectives

(3) If $d_1 > c_2$, Reject the whole lot. Detail the Lot 100%, replacing all bad items by good ones.

(4) If $c_1+1 \le d_1 \le c_2$, take a Second Sample of size n_2 from the Remaining Lot.

(5) If $d_1+d_2 \le c_2$, Accept the lot, replacing defective items by standard ones.

(6) If $d_1+d_2 > c_2$, Reject the whole lot. Inspect the rejected lot 100%, replacing all the defective items by good one.

Dodge and Romig obtained the most economical Double sampling plans after providing adequate protection to producer and consumer s.t.

(i) Average Total Inspection is minimum, and

(ii) the prob^t of acceptance on the basis of first sample is same as the prob^t of acceptance on the basis of second sample.

O.C. Curve of double Sampling plan ⇒

The Lot will be Accepted under the following mutually exclusive ways.

(i) $0 \le d_1 \le c_1$

(ii) $d_1 = c_1+1$; $d_2 \le c_2 - c_1 - 1$

(3) $d_1 = c_1 + 2$; $d_2 \le c_2 - c_1 - 2$

$\vdots$

$d_1 = c_2$; $d_2 = 0$

Hence, by addition theorem of prob^t, the prob^t of acceptance for a lot of Incoming quality p is given by.

$$P_a(p) = \sum_{x=0}^{c_1} g(x,p) + \sum_{y=0}^{c_2-x} \sum_{x=c_1+1}^{c_2} g(x,p)\cdot h(y, p/x)$$

where $g(x,p)$ is the prob^t of finding x defectives in the first sample and $h(y, p/x)$ is the conditional prob^t of finding y defective in the second sample under the condition that x defectives have already appeared in the first sample. Thus.

$$g(x,p) = {}^{Np}C_x \cdot {}^{N-Np}C_{n_1-x} \Big/ {}^{N}C_{n_1}$$

$$h(y, p/x) = {}^{Np-x}C_y \cdot {}^{N-n_1-(Np-x)}C_{n_2-y} \Big/ {}^{N-n_1}C_{n_2}.$$

$$\Rightarrow P_a(p) = \sum_{x=0}^{c_1} {}^{Np}C_x \cdot {}^{N-Np}C_{n_1-x} \Big/ {}^{N}C_{n_1}$$

$$+ \sum_{y=0}^{c_2-x} \sum_{x=c_1+1}^{c_2} \frac{{}^{Np}C_x \cdot {}^{N-Np}C_{n_1-x} \cdot {}^{Np-x}C_y \; {}^{N-n_1-Np+x}C_{n_2-y}}{{}^{N}C_{n_1} \cdot {}^{N-n_1}C_{n_2}}.$$

$$= P_{a_1}(p) + P_{a_2}(p), \quad (\text{say})$$

where $P_{a_1}(p)$ and $P_{a_2}(p)$ are the probabilities of

Acceptance on the basis of first and Second Samples respectively.

Consumer's and Producer's Risk $\Rightarrow$

the consumer's Risk is given by

$P_c = P[\text{Accept a lot of quality } p_t] = P_a(p_t)$

and producer's Risk is given by

$P_p = 1 - P[\text{Accepting a lot of quality } \bar{p}] = 1 - P_a(\bar{p})$.

ASN and ATI of Double Sampling plan ⇒

In an acceptance-rejection double sampling plan, the number of items inspected for a lot is either n_1 or n_1+n_2.

⇒ The Expected Sample size for a decision is given by

$$A.S.N = n_1 P_1 + (n_1+n_2)(1-P_1) = n_1 + n_2(1-P_1).$$

where P_1 is the prob^t of a decision (acceptance or rejection of the lot) on the basis of the first Sample.

However, in a double Sampling acceptance rectification scheme in which rejected Lots are inspected 100%,

The average total inspection (ATI) per lot is given by

$$A.T.I = n_1 P_{a_1}(p) + (n_1+n_2) P_{a_2}(p) + N(1-P_a) \quad —*$$

because

(i) only n_1 items will be inspected if the Lot is accepted on the basis of the first Sample and its prob^t is $P_{a_1}(p)$.

(ii) (n_1+n_2) items will be inspected if the Lot is accepted on the basis of the Second Sample. and its prob^t is $P_{a_2}(p)$.

(iii) The entire Lot of N items will be inspected if the Lot is rejected and the prob^t of this is $1-P_a(p)$.

Since $P_a(p) = P_{a_1}(p) + P_{a_2}(p)$

$$\Rightarrow P_{a_2}(p) = P_a(p) - P_{a_1}(p).$$

we get from *

$$ATI = n_1 P_{a_1}(p) + (n_1+n_2)\left(P_a(p) - P_{a_1}(p)\right) + N\left(1-P_a(p)\right)$$

$$= n_1 + n_2\left(1-P_{a_1}(p)\right) + (N-n_1-n_2)\left(1-P_a(p)\right).$$

Cumulative Sum Control chart (Cu Sum) chart

⇒ A major disadvantage of a Shewhart control chart ($\bar{x}$, R) charts, is that it uses only the Information about the process contained in the last sample observation and it ignores any Information given by the entire sequence of points. This feature makes the Shewhart control chart relatively Insensitive to small process shifts, say, on the order of about 1.5 σ or less. This potentially makes Shewhart control chart less useful. So In this case we use Two very effective alternatives to the Shewhart control chart may be used when small process shifts are of Interest.

(i) Cu Sum control chart
(ii) Exponentially weighted moving Average (EWMA) control chart.

Example ⇒ Consider the data in the following Table, Column (a). The first 20 of these observations were drawn at random from $N(10, 1)$. These observations have been plotted on a shewhart control chart in the following fig. The central line and 3-σ control limits on this chart are at

$U.C.L = 13$

$C.L = 10$

$L.C.L = 7.$

The last 10 observations in column (a) of table were drawn from $N(11, 1)$.

Here we can see that by the Figure that if we apply the traditional shewhart control chart theory all sample points lies within control limits Hence we have no strong evidence that the process is out of control. But there is an Indication of a shift in process level for the last 10 points.

The Reason for this failure, is the relatively

Sample i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.41	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

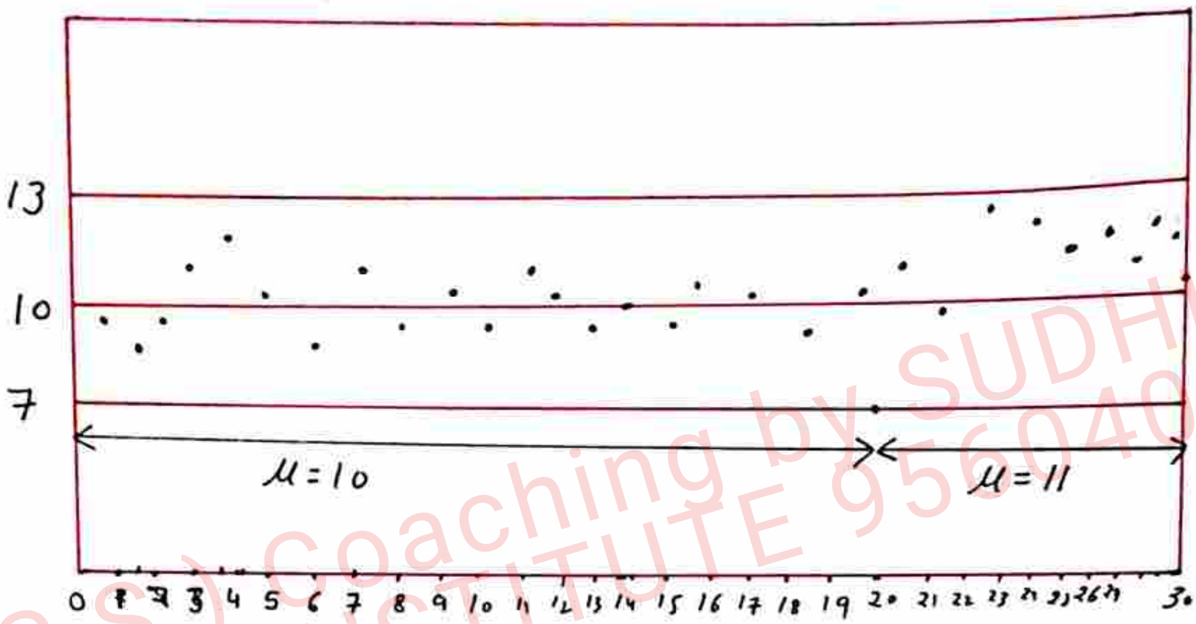


Figure : A Shewhart Control chart for given data.

Small magnitude of the shift.
The Shewhart chart for average is very effective if the magnitude of the shift is 1.5σ or Large.
For smaller shifts, it is not as effective.
The CuSum control chart is a good alternative when small shift are Important.

The CuSum chart directly Incorporates all the Information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a Target Value.

Suppose that sample of size $n \geq 1$ are collected, and $\bar{x}_J$ is the average of the J^{th} sample. Then if μ_0 is the Target for the process mean, the CuSum chart is formed by plotting the quantity

$$C_i = \sum_{J=1}^{i} (\bar{x}_J - \mu_0) \quad \text{---} \; *$$

against the sample no i.

C_i is called the Cumulative Sum up to and including the i^{th} sample.

Because they combine information from several samples, CuSum charts are more effective than shewhart chart for detecting small process shift. Furthermore, they are effective with sample size $n=1$ also.

We note that if the process remains in control at the Target Value μ_0, The Cumulative Sum defined in Eq^n * is a Random walk with mean 0.

If the mean shifts upward to some Value $\mu_1 > \mu_0$ then an upward or positive drift will develop in the Cumulative Sum C_i. Conversely If mean shifts downward to $\mu_1 < \mu_0$, then a downward or negative drift in C_i will develop.

Therefore, If a Significant Trend Develops in the plotted points either upward or downward, we should consider this as evidence that the process mean has shifted, and a search for some assignable cause should be performed.

This theory can be easily demonstrated by using the data in column (a). To apply the cusum in Eq^n * to these observations, we would take $\bar{x}_J = x_J$ (Since $n = 1$) and $\mu_0 = 10$.

therefore CuSum becomes

$$C_i = \sum_{J=1}^{i} (x_J - 10) = (x_i - 10) + \sum_{J=1}^{i-1} (x_J - 10)$$

$$\Rightarrow \quad C_i = (x_i - 10) + C_{i-1} \quad ; \quad C_0 = 0 \text{ (let)}$$

Column (b) contains the differences $(x_i - 10)$ and the cumulative sums are computed in column (c).

The following Figure plots the CuSum from Column (c) of given table.

Note that for the first 20 observations where $\mu = 10$, The cusum tends to drift slowly, in this case maintaining values near 0. However, in the last 10 observations, where the mean has shifted to $\mu = 11$, a strong upward trend Develops.

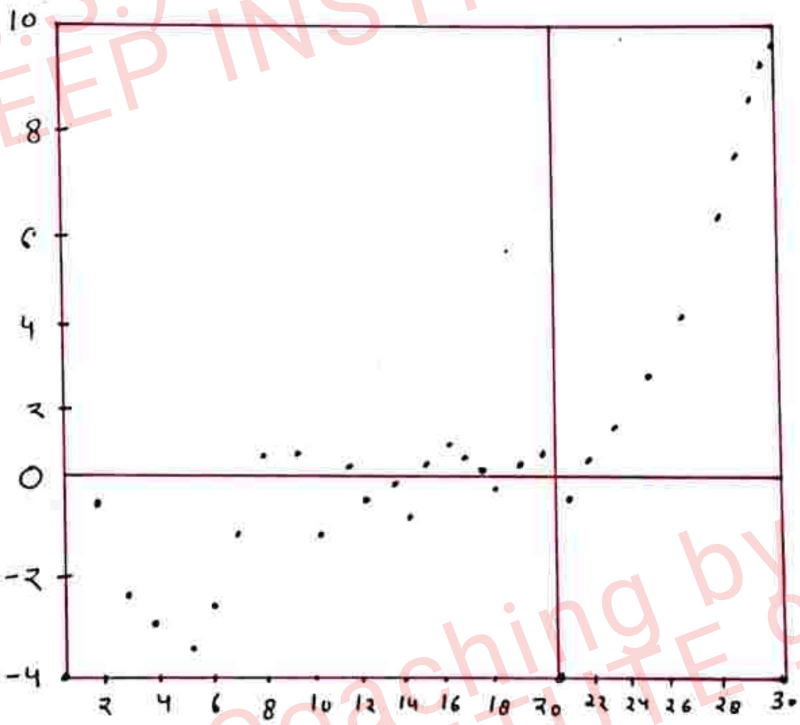


Fig-2 (plot of the cu Sum from column (c))

The CuSum plot in Fig-2 is not a Control chart because it lacks statistical Control limits.

There are Two ways to represent cu Sums,

(i) Tabular (algorithmic) Cu Sum

(ii) V-mask form of the cu Sum.

The Tabular or Algorithmic CuSum for monitoring the process mean ⇒

CuSum may be constructed both for individual observations and for Average of Rational Sub-groups. The case of Individual observations occurs very often in practice.

Let x_i be the i^{th} observation on the process. When the process is in control, x_i has a $N(\mu_0, \sigma)$. σ is Known.

The Tabular CuSum works by accumulating deviations from μ_0 that are above Target with one statistic C^+ and Accumulating deviations from μ_0 that are below Target with another statistic C^-.

The statistics C^+ and C^- are called one-Sided upper and Lower CuSums respectively.

They are calculated as for $n=1$.

$$C_i^+ = \max\{0, x_i - (\mu_0 + k) + C_{i-1}^+\}$$

$$C_i^- = \max\{0, (\mu_0 - k) - x_i + C_{i-1}^-\}$$

$$C_0^+ = C_0^- = 0$$

K is usually called the reference value, and it is often chosen about halfway between the Target μ_0 and the out-of control value of the mean μ_1 that we are interested in detecting quickly.

$$\Rightarrow \quad K = \left| \frac{\mu_1 - \mu_0}{2} \right|$$

"If either C_i^+ or C_i^- exceed the Decision Interval H, ($H = 5\sigma$), the process is considered to be out of control."

If the process is out of control, the action taken on a cusum control scheme is as.

first we calculate quantities N^+ and N^- in table which indicate the number of consecutive periods that the cusums C_i^+ and C_i^- have been nonzero.

Now just count backward from the out of control signal (C_i^+ or $C_i^- > H$) to the time period when the cusum lifted above zero. to find the first period following the process shift.

In situation where an adjustment to some manipulatable variable is required in order to bring the process back to the target value μ_0, it may be helpful to have an estimate of the new process mean following the shift. This can be computed from.

$$\hat{\mu} = \begin{cases} \mu_0 + K + \dfrac{C_i^+}{N^+} & ; \text{ if } C_i^+ > H \\[2ex] \mu_0 - K - \dfrac{C_i^-}{N^-} & ; \text{ if } C_i^- > H. \end{cases}$$

Example $\Rightarrow \mu_0 = 10$; $\sigma = 1$; $\mu_1 = 11$; $K = 0.5$; $H = 5$

		(a)			(b)		
i	x_i	$x_i - (\mu_0 + k)$ $x_i - 10.5$	C_1^+	N^+	$(\mu_0 - k) - x_i$ $9.5 - x_i$	C_1^-	N^-
1	9.45	−1.05	0	0	0.05	0.05	1
2	7.99	−2.51	0	0	1.51	1.56	2
3	11.66	1.16	1.16	1	−2.16	0	0
4	12.16	1.66	2.82	2	−2.66	0	0
5	10.18	−0.32	2.50	3	−0.68	0	0
6	8.04	−2.46	0.04	4	1.46	1.46	1
7	11.46	0.96	1.00	5	−1.96	0	0
8	9.20	−1.3	0	0	0.30	0.30	1
9	10.34	−0.16	0	0	−0.84	0	0
10	9.03	−1.47	0	0	0.47	0.47	1
11	11.47	0.97	0.97	1	−1.97	0	0
12	10.51	0.01	0.98	2	−1.01	0	0
13	9.40	−1.10	0	0	0.10	0.10	1
14	10.08	−0.42	0	0	−0.58	0	0
15	9.37	−1.13	0	0	0.13	0.13	1
16	10.62	0.12	0.12	1	−1.12	0	0
17	10.31	−0.19	0	0	−0.81	0	0
18	8.52	−1.98	0	0	0.98	0.98	1
19	10.84	0.34	0.34	1	−1.34	0	0
20	10.90	0.40	0.74	2	−1.40	0	0
21	9.33	−1.17	0	0	0.17	0.17	1
22	12.29	1.79	1.79	1	−2.79	0	0
23	11.50	1.00	2.79	2	−2.00	0	0
24	10.60	0.10	2.89	3	−1.10	0	0
25	11.08	0.58	3.47	4	−1.58	0	0
26	10.38	−0.12	3.35	5	−0.88	0	0
27	11.62	1.12	4.47	6	−2.12	0	0
28	11.62	1.12	5.28	7	−1.81	0	0
29	11.31	0.81	6.09	8	−1.81	0	0
30	10.52	0.02	6.11	9	−1.02	0	0

The above table presents the tabular cu Sum Scheme. Consider period $i=1$.

$$C_1^+ = \max\{0,\ x_1 - 10.5 + C_0^+\} = 0$$

$$C_1^- = \max\{0,\ 9.5 - x_1 + C_0^-\} = 0.05.$$

Similarly we calculate all C_i^+ and C_i^-. and also we calculate quantities N^+ and N^-.

The Cu Sum calculations in table show that the upper side cu sum at period 28 is

$$C_{28}^+ = 5.28.$$

Since this is the first period at which $C_i^+ > H = 5$ we conclude that the process is out of control at point 28.

The tabular cu sum also indicates when the shift probably occurred.

The counter N^+ records the number of consecutive periods since the upper-side cu sum C_i^+ rose above the value of zero.

Since $N^+ = 7$ at period 28, we would conclude that the process was last in control at period $28 - 7 = 21$

So the shift likely occurred between period 21 and 22.

Now we would estimate the new process Average as

$$\hat{\mu} = \mu_0 + K + \frac{C_{28}^{+}}{N^{+}} = 10 + 0.5 + \frac{5.28}{7} = 11.25$$

The Standardized CuSum ⇒

Many users of the CuSum prefer to standardize the variable x_i before performing the calculations. Let $y_i = \frac{x_i - \mu_0}{\sigma}$

be the standardized value of x_i. Then the standardized cusums are defined as follows.

$$C_i^+ = \max\{0, y_i - k + C_{i-1}^+\}$$

$$C_i^- = \max\{0, -k - y_i + C_{i-1}^-\}$$

$$C_0^+ = C_0^- = 0$$

One-Sided CuSum ⇒

We have focused primarily on the Two-sided CuSum. Note that the tabular procedure is constructed by running two one-sided procedures, C_i^+ and C_i^-. There are situations in which only a single one-sided CuSum procedure is useful.

Sometime it is also possible to design CuSum that have different sensitivity on the upper and lower side. This could be useful in situations where shifts in either direction are of Interest, but shifts above the Target (say) are more critical than shifts below the Target.

It is possible to construct cusum control charts for monitoring process variability.

Let $x_i \sim N(\mu_0, \sigma^2)$. The standardized value of x_i is $y_i = \dfrac{x_i - \mu_0}{\sigma}$.

statistician suggests creating a new standardized quantity

$$V_i = \frac{\sqrt{|y_i|} - 0.822}{0.349}$$

and suggests that the V_i are sensitive to variance changes.

V_i, In control distribution, is approximately $N(0,1)$, Two one-sided standardized deviation cusum can be established as follows.

$$S_i^+ = \max\{0, V_i - K + S_{i-1}^+\}$$

$$S_i^- = \max\{0, -K - V_i + S_{i-1}^-\}$$

$$S_0^+ = S_0^- = 0$$

K and H are selected as in the cusum for controlling the process mean.

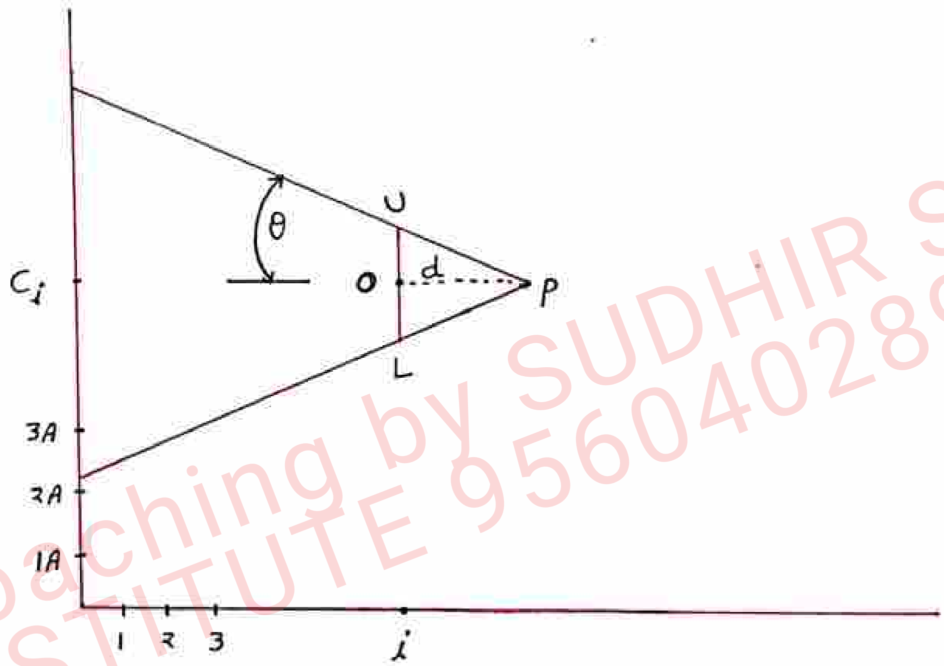
The V-Mask procedure ⇒

An alternative procedure to the use of a tabular CuSum is the V-mask control scheme.

The V-mask is applied to successive values of the CuSum statistic.

$$C_i = \sum_{J=1}^{i} y_J = y_i + C_{i-1}, \quad y_i = \frac{x_i - \mu_0}{\sigma}$$

A Typical V-mask is shown in the following Fig.



The decision procedure consists of placing the V-mask on the cumulative sum control chart with the point O on the Last Value of C_i and the line OP parallel to the Horizontal axis.

If all the previous Cumulative Sums, $C_1, C_2, \ldots C_i$

lie within the Two arms of the V-mask, the process is in control.

If any of the cumulative sum lie outside the arms of the mask, the process is considered to be out of control.

The performance of the V-mask is determined by the lead distance d and the Angle θ.

The tabular CuSum and the V-mask scheme are Equivalent if

$$K = A \cdot \tan\theta$$

and $$H = A \cdot d \cdot \tan\theta = d \cdot K.$$

In these Two Equations, A is the Horizontal distance on the V-mask plot between successive points in Terms of Unit distance on the Vertical Scale.

For example, to construct a V-mask, let $K = \frac{1}{2}$ and $H = 5$, and let $A = 1$

So $$\frac{1}{2} = 1 \cdot \tan\theta \Rightarrow \theta = 26.57^\circ$$

and $$5 = d \cdot \left(\frac{1}{2}\right) \Rightarrow d = 10.$$

that is, the lead distance be 10 Horizontal plotting positions, and $\theta = 26.57^\circ$.

NOTE ⇒ Johnson has suggested a method for designing the V-mask; that is, selecting d and θ, as.

$$\theta = \tan^{-1}\left(\frac{\delta}{2A}\right) \quad \text{and} \quad d = \left(\frac{2}{\delta^2}\right) \cdot \log\left(\frac{1-\beta}{\alpha}\right)$$

where 2α is the greatest allowable probability of a signal when the process mean is on target (a false alarm) and β is the probability of not detecting a shift of size δ.

If β is small then $d \cong -2\frac{\log(\alpha)}{\delta}$

Disadvantages and problems with V-mask Scheme ⇒

The biggest problem with the V-mask is the ambiguity (अस्पष्टता) associated with α and β in the Johnson design procedure.

The Exponentially weighted Moving Average Control chart $\Rightarrow$

It is easier to set up and operate than CuSum.

The Exponentially weighted moving Average (EWMA) is defined as

$$Z_i = \lambda x_i + (1-\lambda) Z_{i-1} \quad \text{where } 0 < \lambda \le 1$$
$$\text{and } Z_0 = \mu_0.$$

To demonstrate that the EWMA Z_i is a weighted average of all previous sample means (or values for $n=1$), as

$$Z_i = \lambda x_i + (1-\lambda) Z_{i-1} = \lambda x_i + (1-\lambda)[\lambda x_{i-1} + (1-\lambda) Z_{i-2}]$$
$$= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 Z_{i-2}.$$
$$\vdots$$
$$= \lambda \sum_{J=0}^{i-1} (1-\lambda)^J x_{i-J} + (1-\lambda)^i Z_0$$

The weight $\lambda(1-\lambda)^J$ decrease geometrically with the age of observations. also the Total weight is 1. i.e

$$\sum_{J=0}^{i-1} \lambda(1-\lambda)^J + (1-\lambda)^i = \lambda\left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)}\right] + (1-\lambda)^i = 1$$

Because these weights decline geometrically when connected by a smooth curve, the EWMA is sometimes called a Geometric moving Average (GMA).

The EWMA is used extensively in time series modeling and in forecasting.

Since the EWMA can be viewed as a weighted average of all past and current observations, it is very Insensitive to the normality assumption. It is therefore an Ideal control chart to use with Individual observations.

If the observations x_i are Independent random Variables with Variance σ^2, then the $V(Z_i)$ is

$$\sigma_{Z_i}^2 = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

Therefore, the EWMA control chart would be constructed by plotting Z_i versus the sample number i.

The center line and control limits (not necessarly line) for the EWMA control chart are as follow

$$U.C.L = \mu_0 + L\cdot\sigma\cdot\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$\text{central line} = \mu_0$$

$$L.C.L = \mu_0 - L\cdot\sigma\cdot\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$L \rightarrow$ width of the control limits. which is choosen by some defined rule.

Since $[1-(1-\lambda)^{2i}] \to 1$ as i is Large.

so If i is large the control limits will approach steady-state Values given by

$$U.C.L = \mu_0 + L\cdot\sigma\cdot\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$L.C.L = \mu_0 - L\cdot\sigma\cdot\sqrt{\frac{\lambda}{(2-\lambda)}}$$

Decision Rule $\Rightarrow$ If Any of the observation or Sample mean $(n>1)$. lies outside the Control limits, we conclude that the process is out of control.

The Moving Average Control chart ⇒

Another Type of Time-weighted control chart based on a Simple, Unweighted Moving Average may be of Interest.

Suppose that Individual observations have been collected, and let $x_1, x_2, \ldots$ denote these observations.

The moving Average of span w at time i is defined as

$$M_i = \frac{x_i + x_{i-1} + x_{i-2} + \cdots + x_{i-w+1}}{w}$$

that is, at time period i, the oldest observations in the moving Average set is dropped and the newest one added to the Set.

The Variance of the moving Average M_i is

$$V(M_i) = \frac{1}{w^2} \sum_{j=i-w+1}^{i} V(x_j) = \frac{\sigma^2}{w} \qquad [\sigma^2 = V(x_j)]$$

therefore, if μ_0 denotes the target Value of the mean used as the center line of the control chart, then 3σ Control limits for M_i are.

$$U.C.L = \mu_0 + \frac{3\sigma}{\sqrt{w}}$$

$$L.C.L = \mu_0 - \frac{3\sigma}{\sqrt{w}}$$

The control procedure would consist of calculating the new moving Average M_i as each observation x_i becomes available, plotting M_i on a control chart with upper and Lower control limits. and concluding that the process is out of control if M_i exceeds the control limits.

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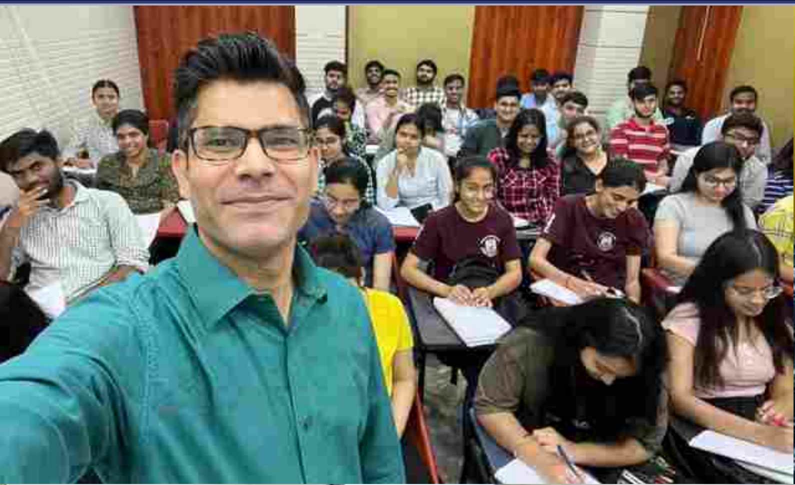

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